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# The Grey Paradox: How Oil Owners Can Benefit From Carbon Regulation.

Renaud Coulomb \*    Fanny Henriet †

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## Abstract

This paper studies how oil owners can benefit from carbon taxation. We build a Hotelling-like model with three energy resources: oil (exhaustible, polluting), coal (non exhaustible, very polluting) and solar energy (non exhaustible, non polluting). The  $CO_2$  concentration must be kept under a carbon ceiling. The optimal extraction path is decentralized by a tax on emissions, and tax revenues are not redistributed. We characterize the different extraction paths. We focus on the case where both oil and coal are extracted and oil gets exhausted. When oil is cheaper to extract than coal, if oil is sufficiently scarce, or if the extraction cost of oil is close enough to the extraction cost of coal or if its pollution content is low enough, or if the demand elasticity is low enough, the profits of oil owners will increase when the carbon regulation is tightened. When oil is more expensive to extract than coal, and both resources are used and oil exhausted, tightening the carbon regulation increases the oil profits.

*Keywords:* Optimal Taxation, Carbon Regulation, Global Warming, Nonrenewable Resources, OPEC, Fossil Fuels, Energy Markets.

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# 1 Introduction

In 2009,  $CO_2$  from energy production represented 65% of greenhouse gas emissions and fossil fuels accounted for 81% of the world energy supply IEA (2011). Taxing carbon emissions have different effects on the profits of fossil fuels owners depending on the characteristics of the fossil fuels they own. Main fossil fuels, coal, gas and oil are marked out by their recoverable reserves, their pollution content, as shown in Table 1, and their delivery cost that varies depending on their use (sector, location, technology) and their extraction cost. This paper studies the impact of tightening a carbon cap over the  $CO_2$  concentration on oil owners profits when energy resources are optimally extracted and the optimum is decentralized by a tax on  $CO_2$  emissions. More generally, we look at how the profits of energy resources owners are affected by optimal carbon taxation.

The Organization of Petroleum Exporting Countries (OPEC) claims compensation for expected losses in oil export revenues due to world-wide or local carbon taxation policy. Two primary reasons may explain why tightening carbon taxation would negatively affect oil revenues: first, oil consumption may decrease in carbon-regulated countries, second, for a given after-tax oil price, carbon taxation transfers rents from oil countries to carbon-regulated countries if tax revenues are not redistributed to oil countries. We show in that paper that profits of oil owners may increase due to optimal carbon taxation even if tax revenues are not redistributed to oil countries. Two characteristics of oil lay behind this result: its exhaustibility and its relatively low pollution content compared to other fossil fuels like coal. Because oil is exhaustible and likely to be exhausted unless the carbon regulation is very strict, the cumulative oil consumption until exhaustion is unchanged in spite of carbon regulation. Secondly, since oil is less polluting than its close competitors, its after-tax price may increase more than the carbon tax, that will rise up oil profits.

Fossil fuels reserves are spread very unequally in the world as shown in Table 2. More than 57% of conventional proved gas reserves are located in Russia, Iran or Qatar. Almost

70% of crude oil proven reserves are located in only six countries: Canada, Saudi Arabia, Iran, Iraq, Kuwait, and United Arab Emirates. Around 75% of recoverable coal reserves are located in only five countries: the USA, Russia, China, Australia and India. To the noticeable exception of Russia, countries having the largest endowments in oil or gas have low endowments in coal. Carbon taxation raise important issues concerning redistribution of wealth among countries. If carbon taxation lowers profits of coal owners but increases profits of oil owners, some countries like OPEC countries may see their wealth increased by carbon taxation contrary to countries with large coal endowments. We focus only on the redistributive aspects driven by changes in energy prices due to carbon taxation. However, benefits or losses from reducing emissions for each country depend also on country-specific vulnerability to climate change, country-specific mitigation costs, the global mitigation effort and country-specific contribution to it.

We build a Hotelling-like model where the  $CO_2$  concentration must be kept under a carbon ceiling. This paper follows modelization introduced by Chakravorty et al. (2006). The social planner model has the following features. Utility comes from three perfect substitute sources of energy: an exhaustible polluting resource (let us say 'oil' or 'conventional oil'), a non-exhaustible strongly polluting resource (let us say 'coal') and a non-exhaustible clean resource (let us say 'solar'). Each resource is distinguishable according to its carbon content and its extraction cost. Resources owners are in perfect competition. The regulation takes the form of a carbon cap over the atmospheric carbon stock. This threshold can be considered as an exogenous constraint, for instance stemming from a Kyoto-like Protocol, or as the first-best carbon policy if the damage function can be approximated by a binary-convex damage function with nil marginal damage when the  $CO_2$  concentration is kept under or equal to the threshold and infinite otherwise. We assume no natural decay of carbon. Resources extraction follows Herfindahl's principle (Herfindahl (1967)): a resource cannot be used before a cheaper-to-extract resource. However, a cheaper resource may not be exhausted if this resource is too polluting. The social planner seeks to maximize the consumer surplus taking account of the oil scarcity constraint and the carbon cap constraint.

To implement his optimal policy, the social planner has a soft power: he can put a carbon tax on  $CO_2$  emissions but cannot forbid the use of a particular resource or set a specific tax or a quota for each different resource. When both oil and coal are used and oil gets exhausted (relevant case), a unique carbon tax path allows to decentralize the equilibrium.

The Hotelling-like model we use is close to Chakravorty et al. (2006). Contrary to Chakravorty et al. (2006), our model includes several polluting resources to be able to point out the increase of oil profits due carbon taxation. Our model shares common features with the model described in Chakravorty et al. (2008) but questions addressed are different. Chakravorty et al. (2008) analysis the optimal order of extraction of two polluting resources under a carbon cap regulation. They found that the most polluting resource may be used prior to the less polluting resource to benefit from natural dilution which increases with the atmospheric stock i.e "least cost first" principle (Herfindahl (1967)) no more holds. Our paper focuses on the impact of profit of oil owners. In our main model, we assume in a first time that only one polluting resource is exhaustible and consider that natural dilution can be negligible to simplify the demonstration. This assumption allows us to simplify the extraction order: a more expensive before-tax resource cannot be extracted before a less expensive before-tax resource following Herfindahl principle (Herfindahl (1967)).

In our model, the value of the carbon ceiling only impacts the use of a resource, that can stop or start to be used due to carbon regulation but does not impact the potential order of extraction that follows the ordering of extraction costs.

As in Chakravorty et al. (2006), Chakravorty et al. (2008), fossil fuels owners are in perfect competition. Oil market has been modeled in various ways throughout economic literature. A large empirical literature has tried to determine which market structure explains the best the dynamics of the oil price. Broadly speaking, this literature can be divided into two groups: a stream arguing that OPEC has a power to influence market price (cartel, dominant firm model, target model) and a second stream where prices are explained either by property rights, scarcity or political events. Reviewing the fourth main assumptions

(cartel, competitive, the target revenue model, the property rights explanation), Griffin (1985) estimates that market-sharing cartel model explains relatively well OPEC behavior. Jones (1990) and Dahl & Yucel (1991) also support the cartel hypothesis. Other papers find that OPEC manipulates prices only over some periods of time, and that increasing prices of 1974-1980 are not due to an OPEC price manipulation (Loderer (1985)). Several studies (Ezzati (1976), MacAvoy (1982) and Verleger (1982)) explain oil prices changes in a competitive model.

The optimum is decentralized by a tax on  $CO_2$  emissions. A particular path of emissions can be decentralized either by a carbon tax or by carbon quotas, as long as the price dynamics of carbon-emitting resources is well understood. In our setting, a price instrument and a quantity instrument are similar from the efficiency point of view since benefits and costs of mitigation actions are known without uncertainty <sup>1</sup>. However, the chosen instrument and its modalities of use impact differently profits of fossil fuels owners on energy markets. A carbon tax without abatement or auctioned exchangeable quotas without redistributing carbon revenues are commonly presented as the worst instruments for the profits of fossil fuels owners. Redistributive aspects of the instruments have received few attention in the debate over climate change regulation whereas in the taxation literature the rent seeking takes an important place. The literature has focused on capturing rents from fossil fuels producers thanks to taxation of externalities (Liski & Tahvonen (2004), Bergstrom (1982)) or using tariffs (Brander & Djajic (1983)). Our paper shows that in some cases taxing externalities increases the profits of some fossil fuels owners.

The Grey Paradox <sup>2</sup> can be expressed in the following way: the profits of oil producers

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<sup>1</sup>For a comparison of both price and quantity instruments with uncertainty in a general framework, one can refer to Weitzman (1974) and its application to the climate change problem is Pizer (1997).

<sup>2</sup>Let us note that in spite of their close names, the phenomenon we describe has nothing to do with the Green Paradox presented by Sinn (2008). The Green Paradox states that subsidizing a renewable backstop (solar, wind energy, nuclear power) brings forward the date at which fossil fuels get exhausted if oil is Hotelling-priced, or if oil producers have a power market, and consequently exacerbates global warming. In such a case, profits of fossil fuels owners are reduced when the clean energy resource price is lowered. On the contrary, in our framework, profits of fossil fuels owners do not necessarily decrease, and emissions are lowered when the carbon cap policy is tightened.

may increase thanks to (optimal) carbon taxation. Without any compensation scheme, introducing a carbon regulation in the form of a carbon tax or exchangeable quotas will affect differently resources owners depending on the carbon content of each resource, but also on their delivery costs and their relative abundance. We fully characterize all the different extraction paths. We study how oil profits change when the ceiling is lowered in the case where both oil and coal are extracted and oil gets exhausted. Let us remark first that when switching from one resource to another, prices of coal and oil must be equal. When tightening the carbon regulation, the carbon tax increases. For a given increase of the carbon tax, the tax paid per unit of coal is larger than the tax paid per unit of oil, due to the difference in pollution contents. It comes that profits of oil owners at the date of switch must increase to keep prices equal. When oil is used before coal (cheaper to extract than coal, the date of switch from oil to coal is postponed, so that the effect on profits is undetermined in the general case. When oil is used before coal, if oil is scarce enough or if the extraction cost of oil is close enough to the extraction cost of coal, tightening the carbon regulation increases the profits of oil owners. Indeed, if the resource is relatively scarce, the switch from oil to coal occurs early. To compensate the increase of the price of coal, profits must increase early. If the oil extraction cost is close enough to the coal extraction cost, tightening the carbon regulation will lead to increase the comparative advantage of oil in terms of pollution. Indeed with similar extraction costs, coal and oil would be perfect substitutes if no carbon regulation was implemented. If oil is sufficiently less polluting than coal, oil owners will benefit from a tighter carbon regulation. If the elasticity of demand is low enough, tightening the carbon ceiling increases oil profits. If the energy demand is concave or linear, oil profits cannot exhibit a U-shape when the carbon ceiling is strengthened. If oil is used after coal (oil is more expensive to extract) and exhausted, tightening the carbon regulation reduces coal consumption, lets global oil consumption unchanged and increases the profits of oil owners.

Persson et al. (2007) use an empirical model to quantify the impact of carbon taxation on oil profits assuming that oil producers are in perfect competition and maximize their



profits over an intertemporal horizon. Johansson et al. (2009) assume that OPEC producers play strategically as a dominant firm in the transport sector facing only fringe producers. Contrary to previous empirical literature, they found that oil profits increase due to carbon regulation. However, no proper theoretical model is built to analyze this phenomenon.

The remainder of this paper is organized as follows. Section 2 presents the main model. Section 3 presents the results over the effects of carbon regulation on the profits of oil owners. Section 4 concludes.

Table 1: Estimated reserves and pollution contents of fossil fuels

<b>Fossil fuels</b>	<b>Reserves (MBtu)</b>	<b>Pollution content (kgCO<sub>2</sub>/MBtu)</b>
Coal	2.32606E+13	103.54
Oil	7.37308E+12	70.02
Gas	6.37507E+12	52.03

IEA (2008), for oil and gas, reserves reported by *Oil and Gas Journal*, MBtu=million of British thermal units.

## 2 The three-resource and one-sector economy: the model

### 2.1 Assumptions and notations

We consider that utility comes from energy consumption. Three different energy resources, perfect substitutes in demand, are available: an exhaustible polluting resource (let us say 'oil') in quantity  $X_o^0$ , a non exhaustible strongly polluting resource (let us say 'coal') footnoteSee der Ploeg & Withagen (2011) for a similar setting oil and in addition with a dirty and clean backstop. and a non exhaustible clean resource (let us say 'solar'). Resources  $o, c, s$  respectively stand for 'oil', 'coal' and 'solar':  $u(x_o(t) + x_c(t) + x_s(t))$ . We write  $D(\cdot)$ , the decreasing energy demand function. We define  $\theta_i$ , for  $i = \{o, c, s\}$  the pollution content

Table 2: Share of fossil fuels reserves by country and world region in 2008.

	Coal	Crude	Natural gas
<b>North America</b>	<b>28.4</b>	<b>15.7</b>	<b>5.1</b>
Canada	0.8	13.4	0.9
United States	27.5	1.4	3.9
<b>Central &amp; South America</b>	<b>1.5</b>	<b>8.3</b>	<b>4.2</b>
Venezuela	0.1	6.5	2.7
<b>Europe</b>	<b>8.9</b>	<b>1.1</b>	<b>2.8</b>
<b>Eurasia</b>	<b>26.5</b>	<b>7.4</b>	<b>32.4</b>
Russia	18.3	4.5	27.0
<b>Middle East</b>	<b>0.1</b>	<b>56.3</b>	<b>41.0</b>
Iran	0.1	10.4	15.2
Iraq	.	8.6	1.8
Kuwait	.	7.8	0.9
Qatar	.	1.1	14.6
Saudi Arabia	.	20.1	4.1
United Arab Emirates	.	7.4	3.4
<b>Africa</b>	<b>3.7</b>	<b>8.6</b>	<b>7.9</b>
<b>Asia &amp; Oceania</b>	<b>30.9</b>	<b>2.6</b>	<b>6.7</b>
Australia	8.9	0.1	0.5
China	13.3	1.2	1.3
India	7.0	0.4	0.6
<b>World</b>	<b>100.0</b>	<b>100.0</b>	<b>100.0</b>

Calculus based on the Energy Information Administration data over recoverable coal reserves, proven crude oil reserves and proven natural gas reserves. Estimates of coal reserves for Iraq, Kuwait, Qatar, Saudi Arabia and United Arab Emirates are missing. Only countries having more than 5% of world reserves of at least one fossil fuel are included in this table.

of resource  $i$ : the use of one unit of resource  $i$  leads to  $\theta_i$  units of  $CO_2$ . We assume that  $\theta_o < \theta_c$  and  $\theta_s = 0$ . Writing  $c_i$ , for  $i = \{o, c, s\}$ , the extraction cost of resource  $i$ , we assume that  $c_o < c_s$  and  $c_c < c_s$ . Coal is non exhaustible (or equivalently coal is abundant enough not to be exhausted for the ceiling regulation we consider). Solar is available in infinite quantity at cost  $c_s$ . The initial amount of oil is written  $X_o^0$ . The variation of oil stock is given by:

$$\dot{X}_o(t) = -x_o(t)$$

We assume that the social discount rate is constant and equals  $r$ . The carbon stock, written  $Z(t)$ , must be kept under a threshold  $\bar{Z}$ . This threshold can be considered as an exogenous constraint, for instance stemming from a Kyoto-like Protocol. This type of carbon regulation is closer to first-best carbon regulation than constant tax policy due to the fact that marginal damage are steeply increasing with the carbon stock.<sup>3</sup> Since the dirty backstop is available in infinite quantity, the ceiling is binding for any value of  $\bar{Z}$ . We assume that  $\bar{Z} > Z^0$ . There is no natural decay of carbon. Thus the variation of the carbon stock through time is simply given by:

$$\dot{Z}(t) = \theta_o x_o(t) + \theta_c x_c(t)$$

To implement his optimal policy, the social planner has a soft power: he can put a carbon tax on  $CO_2$  emissions but cannot forbid the use of a particular resource or set a specific tax, a quota for each different resource. This carbon tax can be paid by consumers (demand side) or by fossil fuels providers (extraction side). Resources owners are in perfect competition.

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<sup>3</sup>All results found below hold with a constant carbon tax. With a constant carbon tax rather than a carbon ceiling, ordering the extraction would become simpler. Writing  $\mu$ , the unitary tax, the oil price and the coal price respectively become:  $c_o + \lambda^0 e^{rt} + \theta_o \mu$  and  $c_c + \theta_c \mu$ . Because, backstop prices are constant through time, for a given set of parameters, only one backstop is used through time (coal or solar, coal or gas etc.). Thus condition to get both coal and oil used (and oil exhausted) write  $c_o + \theta_o \mu < c_c + \theta_c \mu < c_s$  and  $\theta_o X_o^0 < \bar{Z} - Z^0$ . The solution must satisfy:

$$\begin{aligned} \int_0^T D(c_o + \lambda^0 e^{rt} + \theta_o \mu) dt &= X_o^0 \\ c_o + \lambda^0 e^{rT} + \theta_o \mu &= \text{Min}(c_s; c_c + \theta_c \mu). \end{aligned}$$

## 2.2 The Welfare maximization program

The social planner seeks to find the extraction  $\{x_o(t), x_c(t), x_s(t)\}$  which maximizes the net discounted social surplus under the environmental constraint:

$$\int_0^{\infty} e^{-rt} \left( u(x_o(t) + x_c(t) + x_s(t)) - c_o x_o(t) - c_c x_c(t) - c_s x_s(t) \right) dt$$

s.t.

$$\dot{X}_o(t) = -x_o(t)$$

$$\dot{Z}(t) = \theta_o x_o(t) + \theta_c x_c(t)$$

$$Z(t) \leq \bar{Z}$$

$$X_o(t), x_i(t) \geq 0$$

with  $Z^0, X_o^0$  given.

Writing  $\lambda_o(t)$  the shadow value of the remaining stock of oil  $X_o(t)$  and  $\mu(t)$  the shadow cost of the pollution stock  $Z(t)$ , Transversality conditions are given by:

$$\lim_{t \rightarrow \infty} \lambda_o(t) e^{-rt} X_o(t) = 0 \quad (2.1)$$

$$\lim_{t \rightarrow \infty} \mu(t) e^{-rt} Z(t) = 0 \quad (2.2)$$

Equation 2.1 simply states that the exhaustible resource must be exhausted in the long run if the scarcity rent is positive.

### 2.3 First Order Conditions

We define the current value Hamiltonian:

$$\begin{aligned}
 H(t) = & u(x_o(t) + x_c(t)) - c_o x_o(t) - c_c x_c(t) \\
 & - \lambda_o(t) x_o \\
 & - \mu(t) (\theta_o x_o(t) + \theta_c x_c(t))
 \end{aligned}$$

With the following slackness conditions:

$$\nu(t) \geq 0, \quad \text{and} \quad \nu(t)(\bar{Z} - Z(t)) = 0 \quad (2.3)$$

$$\beta(t) \geq 0 \quad \text{and} \quad \beta(t)X_o(t) = 0 \quad (2.4)$$

$$\epsilon(t) \geq 0, \quad \text{and} \quad \epsilon(t)x_o(t) = 0 \quad (2.5)$$

For any control  $\{x_o(t), x_c(t)\}$  there exist co-state variables  $\lambda_o(t), \mu(t)$ , that must satisfy following conditions together with Transversality conditions and slackness conditions:

$$\dot{\lambda}_o(t) = r\lambda_o(t) - \frac{\partial H(t)}{\partial X_o(t)} \iff \dot{\lambda}_o(t) = r\lambda_o(t) + \beta(t) \quad (2.6)$$

$$\dot{\mu}(t) = r\mu(t) - \frac{\partial H(t)}{\partial Z(t)} \iff \dot{\mu}(t) = r\mu(t) + \nu(t) \quad (2.7)$$

$$\frac{\partial H(t)}{\partial x_o(t)} = 0 \iff p_o(t) = c_o + \lambda_o(t) + \theta_o \mu(t) \quad (2.8)$$

$$\frac{\partial H(t)}{\partial x_c(t)} = 0 \iff p_c(t) = c_c + \theta_c \mu(t) \quad (2.9)$$

$$\frac{\partial H(t)}{\partial x_s(t)} = 0 \iff p_s(t) = c_s \quad (2.10)$$

The co-state variable  $\lambda_o(t)$  represents the current value of the scarcity rent of the exhaustible resource. As shown in Hotelling (1931), it increases at rate  $r$ : the discounted net marginal surplus of extraction must be constant. Along the optimal path, extracting a supplementary unit must be equivalent to saving it for a latter use. Writing  $\lambda_o^0 \equiv \lambda_o(0)$ , it

comes that:

$$\lambda_o(t) = \lambda_o^0 e^{rt}.$$

The co-state variable  $\mu(t)$  represents the current value of the shadow cost of marginal pollution. It exhibits a familiar pattern driven by the ceiling-shaped carbon regulation. If the ceiling does not bind but will bind, the pollution cost increases at the rate of the discount rate. The intuition behind this result is similar to Hotelling rule since emissions can be seen as extracting clean air from a reservoir defined by  $\bar{Z} - Z(t)$ . With positive constant natural dilution, this result is unchanged. Writing  $\mu^0 \equiv \mu(0)$ , it comes that:

$$\mu(t) = \mu^0 e^{rt}.$$

The optimal price of oil is simply the sum of its extraction cost, its pollution cost and its scarcity rent by equation 2.8. The optimal price of coal is simply the sum of its extraction cost and its pollution cost by equation 2.9. The unitary pollution cost of carbon is independent from the source it comes from, but the pollution costs of each resource are different since coal is more pollution than coal for a given amount of energy.

Finding the optimal extraction path requires to determine the initial scarcity rent,  $\lambda_o^0$ , the initial shadow cost of pollution  $\mu^0$  and the date of switch from one fossil fuel to another,  $t_1$ , and the date the ceiling is reached,  $\underline{t}$ , such that taking account of the dynamics of the prices, the scarcity rent and the carbon tax (expressed by equations 2.6, 2.7, 2.8, 2.9), the solution verifies that the energy price is continuous through time (this implies that price is continuous at the switch dates from one resource to another,  $t_1$ , and  $\underline{t}$ ). If oil is exhausted, it gets exhausted when its price reaches the price of the following resource in the extraction order (if oil is used in first position, coal succeeds to oil at time  $t_1$ , if oil is used in second position, solar succeeds to oil, at time  $\underline{t}$ ), and the backstop starts to be used exactly when the carbon ceiling starts to bind, at time  $\underline{t}$  when cumulated emissions equal  $\bar{Z} - Z^0$ . These three types of conditions represent necessary and sufficient conditions to get an optimal

control policy.

Decentralizing the optimal extraction path requires to implement a carbon tax. We show below that when both oil and coal are used and oil gets exhausted, a unique carbon tax exists that allows to decentralize the equilibrium (Lemma 1). This tax must equal the shadow cost of pollution,  $\mu(t)$ . In that case, if oil gets exhausted current marginal profit writes  $\lambda_o(t)$ .

## 2.4 Ordering resources extraction: the "least cost first" principle.

Without natural absorption and without CCS, we stop using fossil fuels once the  $CO_2$  concentration reaches the ceiling at date  $\underline{t}$ , thus the date the ceiling binds corresponds to the date of switch to the clean backstop. Without natural absorption, the maximum amount of pollution put in the atmosphere is fixed and equals  $\bar{Z} - Z^0$ .

With equal extraction costs, there is no particular order of extraction, energy prices are equal through the whole path if both resources are used. Hereafter it is assumed that extraction costs are different. From price equations 2.8 and 2.9, it comes that there is no stop-and-go in the use of a polluting resource and no joint use. A resource whose extraction cost is higher than another resource cannot be used before that resource following the Herfindahl principle (Herfindahl (1967)). Indeed, without natural dilution of  $CO_2$ , the scarcity rent and the pollution cost increase at the same rate  $r$ . If  $c_o < c_c$  and  $\lambda_o^0 + \theta_o \mu^0 > \theta_c \mu^0$ , it would imply that  $\forall t, p_c(t) < p_o(t)$  and only coal would be used whereas oil would be cheaper and less polluting than coal.

Finally, among the different paths of extraction (see Table 3), only cases where both resources are used and oil gets exhausted (Cases 1.A and 2.A) are relevant to study the Grey Paradox. It follows that the analysis only focuses on these cases. However, conditions over parameters to get these cases are examined.

Table 3: The different extraction paths with two polluting resources.

Case	Coal position	Oil	Coal
Case 1.A	$c_c > c_o$	used, exh.	used
Case 1.B (Not relevant)	...	used	used
Case 2.A	$c_c < c_o$	used, exh.	used
Case 2.B (Not relevant)	...	used	used
Case 2.C (Not relevant)	...	not used	used
Case 2.D (Not relevant)	...	used	not used

"..." means that the value of that cell is the same than the value of the cell right above.

## 2.5 Decentralization of the optimal extraction path by using a tax on CO2 emissions

We assume that fossil fuels owners face no threat concerning their property rights over their fossil fuels reserves (See Strand (2010) for a study of the impact of unsecured property rights over extraction). We assume that the tax scheme of the social planner is credible for perfectly foresighted individuals and fossil fuels owners are in perfect competition. The carbon tax can be paid by consumers (demand side) or by fossil fuels providers (extraction side). Both options lead to the same results in this framework.

**Lemma 1.** *If both coal and oil are used and oil gets exhausted, setting the carbon tax to the value of the shadow cost of pollution,  $\mu(t)$ , is the only way to decentralize the optimum.*

In a decentralized economy, writing  $\pi(t)$ , unitary oil profits and  $\tau(t)$  the carbon tax, optimal prices must write:

$$p_o(t) = c_o + \lambda_o(t) + \theta_o \mu(t) = c_o + \pi_o(t) + \theta_o \tau(t) \text{ when } p_o(t) < p_c(t)$$

$$p_c(t) = c_c + \theta_c \mu(t) = c_c + \theta_c \tau(t) \text{ when } p_o(t) > p_c(t) \text{ where } \lambda_o(t) \text{ and } \mu(t) \text{ are defined by}$$

the set of necessary conditions over the continuity of the energy price, oil exhaustion, and cumulative emissions.



A necessary condition to decentralize the optimum is that profits increase at the interest rate, otherwise fossil fuels owners would have an incentive to reallocate the resource extraction to increase their profits. It is clear that setting the carbon tax at  $\mu(t)$  allows to decentralize the optimum. We show that if both resources are used and oil gets exhausted that  $\mu(t)$  is the only carbon tax that allows to decentralize the equilibrium. Since the energy price path is fully determined and unique and profits must increase at rate  $r$ , the carbon tax has specific dynamics. In our model, oil and coal prices net of extraction cost increase at the rate of the social discount rate, thus the tax must increase also at the social discount rate. We assume that both resources are used, and oil gets exhausted. It is clear that the tax cannot be discontinuous when coal is used or when oil is used. When coal is used, by equation 2.9, the carbon tax equals  $\mu(t)$  and increases at rate  $r$ . If the tax is different that  $\mu(t)$  when oil is used, the carbon tax must be discontinuous at the date of switch from one fossil fuel to another. We call that date of switch,  $t_1$ . However, there is no downward jump of the carbon tax at  $t_1$ , since otherwise owners of the resource used in position 1 would have an incentive to postpone their extraction to increase their profits, and similarly there is no upward jump since otherwise, owners of resource used in position 2 could increase their profits by bringing forward their extraction. It comes that the tax is continuous, and thus there is a unique initial value of the tax that allows to decentralize the optimum,  $\mu^0$ .

In other words, the dynamics of the carbon tax in such models is always given, but when both polluting resources are used, the value of the tax when extraction goes from one polluting resource to the other polluting resource is fixed thus the entire tax path is determined.

**Corollary 2.** *When both resources are used and oil gets exhausted, marginal profit at time  $t$  is given by  $\lambda_o(t)$ . Cumulative discounted profits are proportional to the initial unitary profit, the scarcity rent  $\lambda_o^0$ . Profits in value of time 0 write:*

$$\Pi = \int_0^\infty e^{-rt} D(p(t)) \lambda_o(t) dt = \lambda_o^0 X_o^0.$$

If oil is not exhausted, perfect competition among oil owners annihilates profits. When

only one polluting resource is used (Chakravorty et al. (2006)), even if the energy price is well defined through time, there is an infinite number of ways of setting the carbon tax and the scarcity rent to implement the optimal energy price under the conditions that profits increase at rate  $r$  and the energy price is continuous. The social planner can always by setting a tax capture the profits of oil producers. To fully capture profits, he can set the tax equal to the optimal price net of extraction cost.

### 3 Results

#### 3.1 Oil then coal, $c_o < c_c$

If  $c_o < c_c$ , since oil is less polluting than coal, oil is necessarily used. Two cases exist depending on whether oil gets exhausted (coal used) or not (coal not used). Oil gets exhausted and both resources are used iff.  $X_o^0 < \frac{\bar{Z}-Z^0}{\theta_o}$ .

We consider Case 1.A.  $X_o^0 < \frac{\bar{Z}-Z^0}{\theta_o}$ , both oil and coal are used to get to the ceiling. The social planner must set the carbon tax such that oil is exhausted when its price equals coal price, given oil consumption and coal consumption until coal price equals the backstop price when the carbon stock reaches the carbon ceiling. The solution  $\{\lambda_o^0, \mu^0, t_1, \underline{t}\}$ <sup>4</sup> must satisfy:

$$c_o + \lambda_o^0 e^{rt_1} + \theta_o \mu^0 e^{rt_1} = c_c + \theta_c \mu^0 e^{rt_1} \quad (3.1)$$

$$c_c + \theta_c \mu^0 e^{r\underline{t}} = c_s \quad (3.2)$$

$$\int_0^{t_1} D(c_o + \lambda_o^0 e^{rt} + \theta_o \mu^0 e^{rt}) dt = X_o^0 \quad (3.3)$$

$$\theta_o X_o^0 + \int_{t_1}^{\underline{t}} \theta_c D(c_c + \theta_c \mu^0 e^{rt}) dt = \bar{Z} - Z^0 \quad (3.4)$$

Case 1.A is described in Figure 4. Bold curves represent coal and oil prices and the medium curve represent the scarcity rent. The initial prices paths are described by the

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<sup>4</sup>If  $X_o^0 > \frac{\bar{Z}-Z^0}{\theta_o}$ , oil is not fully exhausted and the solution is described by a similar set of equations, excluding coal, and setting oil rent to zero, excluding equation of oil exhaustion (equation 3.3) and equating final price of oil to the backstop price. The level of the carbon tax and the scarcity rent are undetermined. The social planner will take the whole carbon tax revenues for itself, it comes that profits of oil producers are discontinuous and exhibit a downward jumps to zero when  $\bar{Z}$  goes over  $Z^0 + \theta_o X_o^0$

plain curves. Dotted curves represent these price and the scarcity after a decrease of the ceiling. Vertical bold Black lines represent the tax by unit of oil and coal and vertical bold Grey lines represent the oil profits at the date of switch from one resource to another.

Reducing the carbon ceiling increases the carbon tax (effect 1 in Figure 4). Indeed, let assume that this is not the case, it comes that coal price decreases over the whole path. Oil price must also decrease otherwise, global oil consumption would be reduced while coal consumption would be increased, and oil would not be exhausted. However, if oil price decreases, oil gets exhausted earlier. At the new switch date from oil to coal, the coal price is thus lower than the switch price before the carbon ceiling was tightened, thus coal consumption increases. Contradiction. The price at which coal starts to be used increases (effect 2). Indeed let assume that this is the opposite. If the switch date  $t_1$  decreases, the initial energy price must decrease to keep oil exhausted, thus the price at which oil gets exhausted and coal starts to be used, must decrease (effect 3). However, when tightening the carbon constraint by equation 3.4, coal consumption must decrease thus the price at which coal starts to be used must increase. Contradiction. The date the ceiling binds is brought forward when the carbon ceiling is tightened (effect 4).

Tightening the ceiling has two effects on oil profits. The price at which coal starts to be used increases (effect 1), so that value of the tax at the date of switch (long vertical bold black lines),  $\mu^0 e^{rt_1}$  increases. The oil rent at the date oil is exhausted (vertical bold Grey lines) increases as well (effect 3), as it satisfies  $\lambda_o^0 e^{rt_1} = (c_c - c_o) + (\theta_c - \theta_o)\mu^0 e^{rt_1}$ . On the other hand, as the tax increases, the after-tax price of oil increases, the demand at each date decreases so that the date  $t_1$  at which oil is exhausted, is postponed. The final price of oil is higher, the current oil profits when it gets exhausted are higher but oil gets exhausted over a longer period of time. It is not straightforward to see which effect is the larger (effect 5). Let look at a (too) simple case however. Assume that the demand is totally inelastic. Then, the only effect of a carbon tax is not to postpone extraction (the demand remains the same at each date during extraction) but to shorten the length of extraction, that is to say

to bring forward the date at which we switch from coal to the clean backstop. The date  $t_1$  of switch from oil to coal, on the other hand, cannot be moved (as long as  $\bar{Z} - Z_0 > \theta_o X_o^0$ ). So that the second effect described earlier vanishes: when the tax increases, the date  $t_1$  at which oil is exhausted is not postponed. On the other hand, the first effect does not disappear: the date at which coal extraction stops must be brought forward, so that the tax must increase at each date in order to make the price of coal  $c_c + \theta_c \mu^0 e^{rt}$  reach  $c_s$  sooner. As a result, the final before tax price of oil is increased ( $\lambda_o^0 e^{rt_1} = (c_c - c_o) + (\theta_c - \theta_o) \mu^0 e^{rt_1}$ ), as  $t_1$  remains the same, the scarcity rent of oil producers increases.

**Lemma 3.**  $\forall \bar{Z} > \theta_o X_o^0 + Z^0$ ,

$$\frac{d\lambda_o^0}{d\bar{Z}} < 0$$

iff.

$$\frac{D(p(t_1))}{D(p(0))} \frac{\theta_c}{\theta_o} + (\theta_c - \theta_o) \frac{\mu^0}{\lambda_o^0} > 1$$

*Proof.* Differentiating the previous system with respect to  $\bar{Z}$ , it comes that

$\frac{d\lambda_o^0}{d\bar{Z}}$  has the sign of:

$$-\theta_c D(p(t_1)) + \theta_o D(p(0)) + r\theta_c \mu^0 \theta_o \int_0^{t_1} D'(p(t)) e^{rt} dt$$

that can be rewritten, using that  $\dot{p}(t) = r(\lambda_o^0 + \theta_o \mu^0) e^{rt}$ ,

$$-\theta_c D(p(t_1)) + \theta_o D(p(0)) + \theta_c \mu^0 \theta_o \frac{D(p(t_1)) - D(p(0))}{\lambda_o^0 + \theta_o \mu^0}$$

thus finally,  $\frac{d\lambda_o^0}{d\bar{Z}}$  has the sign of:

$$1 - \frac{D(p(t_1))}{D(p(0))} \frac{\theta_c}{\theta_o} - (\theta_c - \theta_o) \frac{\mu^0}{\lambda_o^0}$$

□

Remark that  $(\theta_c - \theta_o) \frac{\mu^0}{\lambda_o^0} > 0$ , then a sufficient condition is that  $\frac{D(p(t_1)) \theta_c}{D(p(0)) \theta_o} > 1$ . When  $D(p(t_1)) = D(p(0))$ , it is always the case that  $\frac{D(p(t_1)) \theta_c}{D(p(0)) \theta_o} > 1$ . This very simple example shows the role of the elasticity of demand on the outcome. Without any more information on this elasticity, one can, however, show that following propositions hold:

**Proposition 4.**  $\exists \epsilon^*$  such that :

$$\left\{ \forall p, -\frac{D'(p)p}{D(p)} \leq \epsilon^* \text{ and } X_o > D(c_s) \right\} \implies \frac{d\lambda_o^0}{dZ} < 0$$

*If the elasticity of demand is small enough, tightening the carbon ceiling increases the profit of oil producers.*

*Proof.* We know that

$$\frac{d\lambda_o^0}{dZ} < 0$$

iff.

$$\frac{D(p(t_1)) \theta_c}{D(p(0)) \theta_o} + (\theta_c - \theta_o) \frac{\mu^0}{\lambda_o^0} > 1$$

By the mean value theorem, there exists a date  $t_i$ , satisfying  $0 \leq t_i \leq t_1$  such that:

$$\begin{aligned} \frac{D(p(t_1))}{D(p(0))} &= 1 + \frac{D'(p(t_i))}{D(p(0))} (p(t_1) - p(0)) \\ &= 1 + \frac{D'(p(t_i))}{D(p(t_i))} p(t_i) \frac{D(p(t_i))}{D(p(0))} \frac{p(t_1) - p(0)}{p(t_i)} \\ &\geq 1 + \frac{D'(p(t_i))}{D(p(t_i))} p(t_i) \frac{p(t_1) - p(0)}{p(t_i)} \end{aligned}$$

But

$$\begin{aligned} \frac{p(t_1) - p(0)}{p(t_i)} &\leq \frac{p(t_1) - p(0)}{p(0)} \\ &\leq \frac{p(t_1) - c_0}{p(0) - c_0} - 1 \\ &\leq e^{rt_1} - 1 \\ &\leq e^{r \frac{X_o}{D(c_s)}} - 1 \end{aligned}$$

So that

$$\frac{D(p(t_1))}{D(p(0))} \geq 1 - \left(-\frac{D'(p(t_i))}{D(p(t_i))}p(t_i)\right)(e^{r\frac{X_o}{D(c_s)}} - 1)$$

If  $\forall p, -\frac{D'(p)}{D(p)}p \leq \frac{1-\theta_o/\theta_c}{e^{r\frac{X_o}{D(c_s)}}-1} \leq \frac{1-\theta_o/\theta_c}{e^r-1} \equiv \epsilon^*$ , then tightening the carbon ceiling increases oil profits.  $\square$

**Proposition 5.**  $\forall \bar{Z} > \theta_o X_o^0 + Z^0, \exists \theta < \theta^*(\bar{Z}) < 1$  such that:

$$\theta_o \leq \theta^* \implies \frac{d\lambda_o^0}{d\bar{Z}} < 0$$

*If the pollution content of oil is low enough, tightening the carbon ceiling increases the profit of oil producers.*

*Proof.* Straightforward from Lemma 3  $\square$

**Proposition 6.**  $\forall \bar{Z} > \theta_o X_o^0 + Z^0, \forall c_c, \exists c^* < c_c$  such that  $\forall c_o$ :

$$c_c \geq c_o \geq c^* \implies \frac{d\lambda_o^0}{d\bar{Z}} < 0$$

*If oil extraction cost is close enough from coal extraction cost, tightening the carbon ceiling increases the profit of oil producers.*

*Proof.* Using Lemma 3, and replacing  $(\theta_c - \theta_o)\frac{\mu_o^0}{\lambda_o^0}$  by  $1 - \frac{(c_c - c_o)}{\lambda_o e^{rt_1}}$ , it comes that  $\frac{d\lambda_o^0}{d\bar{Z}}$  has the sign of:

$$-\frac{D(p(t_1))}{D(p(0))} \frac{\theta_c}{\theta_o} + \frac{(c_c - c_o)}{\lambda_o e^{rt_1}}$$

But  $\lambda_o^0 e^{rt_1} > (\theta_c - \theta_o) \mu^0 e^{rt_1}$ . So that:  $\frac{d\lambda_o^0}{d\bar{Z}} < -\frac{D(c_s)}{D(c_o)} \frac{\theta_c}{\theta_o} + \frac{(c_c - c_o)}{(\theta_c - \theta_o) \mu^0 e^{rt_1}}$ .

But  $(\mu^0 e^{rt_1}, \underline{t} - t_1)$  are defined by :

$$\begin{aligned} \theta_c \int_0^{\underline{t} - t_1} D(c_c + \mu^0 e^{rt_1 + ru}) du &= \bar{Z} - Z_0 - \theta_o X_o \\ \mu^0 e^{rt_1} e^{r(\underline{t} - t_1)} &= c_s - c_c \end{aligned}$$

So that  $\mu^0 e^{rt_1}$  does not depend on  $c_o$  and is strictly positive for any  $\bar{Z}$ . At  $\mu_o e^{rt_1}$  given, as  $-\frac{D(c_s)}{D(c_o)} \frac{\theta_c}{\theta_o} + \frac{(c_c - c_o)}{(\theta_c - \theta_o) \mu^0 e^{rt_1}}$  is continuous with  $c_o$  and decreases with  $c_o$  and is strictly negative for  $c_o = c_c$ , then there exists  $c^*$  such that Proposition 6 holds.  $\square$

**Proposition 7.**  $\forall \bar{Z} > \theta_o X_o^0 + Z^0$ , there exists  $X^*(\bar{Z})$  such that:

$$X_o^0 < X^*(\bar{Z}) \implies \frac{d\lambda_o^0}{d\bar{Z}} < 0$$

*For a resource scarce enough, tightening the carbon ceiling increases the profit of oil producers.*

*Proof.* Demands  $D(t_1)$  and  $D(0)$  are continuous functions of the initial oil stock  $X_o^0$ . Moreover,  $\lim_{X \rightarrow 0} \frac{D(t_1)}{D(0)} = 1$ , as a result,  $\forall \epsilon, \exists X^*$  such that  $X_o^0 < X^* \implies \frac{D(t_1)}{D(0)} \geq 1 - \epsilon$ . Take  $\epsilon < \frac{\theta_c - \theta_o}{\theta_c}$  and the corresponding  $X^*$ , then it is the case that for  $X_o^0 < X^*$ ,  $\frac{d\lambda_o^0}{d\bar{Z}} < 0$   $\square$

*Remark 8.*

$$\lim_{\bar{Z} \rightarrow Z_0 + \theta_o X_o^+} \lambda_o^0 > 0$$

*Proof.* Using equation 3.1, it comes that  $\lambda_o^0 = e^{-rt_1} (c_c - c_o) + (\theta_c - \theta_o) \mu^0$ . Rewriting equation 3.4 into  $X_o^0 \theta_o + \int_{t_1}^{\underline{t}} \theta_c D(c_c + \theta_c \mu^0 e^{rt}) dt = \bar{Z} - Z^0$ , it comes that  $\lim_{\bar{Z} \rightarrow Z_0 + \theta_o X_o^+} (\underline{t} - t_1) = 0^+$ . Using Equation 3.2,  $c_c + \theta_c \mu^0 e^{r\underline{t}} = c_s$  we get  $\lambda_o^0 = e^{-rt_1} (c_c - c_o) + (\theta_c - \theta_o) \frac{c_s - c_c}{\theta_c e^{r\underline{t}}}$ . If  $\lim_{\bar{Z} \rightarrow Z_0 + \theta_o X_o^+} \lambda_o^0 = 0$ , thus  $\lim_{\bar{Z} \rightarrow Z_0 + \theta_o X_o^+} e^{-rt_1} (c_c - c_o) + (\theta_c - \theta_o) \frac{c_s - c_c}{\theta_c} e^{-r\underline{t}} = 0$  Contradiction.  $\square$

**Proposition 9.** *If  $D'' < 0$ ,  $\forall \bar{Z}^1, \bar{Z}^2$  such that,  $\bar{Z}^1 > \bar{Z}^2$ ,*

$$\frac{d\lambda_o^0}{d\bar{Z}} \Big|_{\bar{Z}=\bar{Z}^1} > 0 \implies \frac{d\lambda_o^0}{d\bar{Z}} \Big|_{\bar{Z}=\bar{Z}^2} > 0$$

*For a concave or linear demand function, oil profits cannot exhibit a U-shape when the carbon ceiling is tightened. In other words, when the energy demand is concave or linear, the Grey Paradox cannot occur when making the carbon regulation more stringent, if it does no occur for a less strict carbon regulation.*

*Proof.* From Lemma 3,  $\frac{d\lambda_o^0}{d\bar{Z}}$  has the sign of  $N = -\theta_c e^{rt_1} D(p(t_1)) + \theta_o e^{rt_1} D(p(0)) + r\theta_c \mu^0 e^{rt_1} \theta_o \int_0^{t_1} D'(p(t)) e^{rt} dt$ .  $\frac{d^2 \lambda_o^0}{d\bar{Z} d\bar{Z}}$  has the sign of  $\frac{dN}{d\bar{Z}}$ .

$$\begin{aligned} \frac{dN}{d\bar{Z}} &= -\theta_c D'(p(t_1)) (\lambda_o^0 r e^{rt_1} \frac{dt_1}{d\bar{Z}} + \theta_o \mu^0 r e^{rt_1} \frac{dt_1}{d\bar{Z}} + \theta_o e^{rt_1} \frac{d\mu^0}{d\bar{Z}}) + \theta_o D'(p(0)) \theta_o \frac{d\mu^0}{d\bar{Z}} \\ &\quad + r\theta_c \frac{d\mu^0}{d\bar{Z}} \theta_o \int_0^{t_1} D'() e^{rt} dt + r\theta_c \mu^0 \theta_o \int_0^{t_1} D''() e^{rt} \theta_o \frac{d\mu^0}{d\bar{Z}} e^{rt} dt \\ &\quad + D'(p(t_1)) e^{rt_1} \frac{dt_1}{d\bar{Z}} \\ &= -\theta_c D'(p(t_1)) (\lambda_o^0 r e^{rt_1} \frac{dt_1}{d\bar{Z}} + \theta_o e^{rt_1} \frac{d\mu^0}{d\bar{Z}}) + \theta_o D'(p(0)) \theta_o \frac{d\mu^0}{d\bar{Z}} \\ &\quad + r\theta_c \frac{d\mu^0}{d\bar{Z}} \theta_o \int_0^{t_1} D'() e^{rt} dt + r\theta_c \mu^0 \theta_o \int_0^{t_1} D''() e^{rt} \theta_o \frac{d\mu^0}{d\bar{Z}} e^{rt} dt \end{aligned}$$

Applying integration by parts to  $r \int_0^{t_1} D'(p(t)) e^{rt} dt$ , we get:

$$\begin{aligned} \frac{dN}{d\bar{Z}} &= -\theta_c D'(p(t_1)) (\lambda_o^0 r e^{rt_1} \frac{dt_1}{d\bar{Z}} + \theta_o e^{rt_1} \frac{d\mu^0}{d\bar{Z}}) + \theta_o D'(p(0)) \theta_o \frac{d\mu^0}{d\bar{Z}} \\ &\quad + \theta_c \frac{d\mu^0}{d\bar{Z}} \theta_o \left( D'(p(t_1)) e^{rt_1} - D'(p(0)) - \int_0^{t_1} D''(p(t)) e^{rt} (\lambda_o^0 + \theta_o \mu^0) r e^{rt} dt \right) \\ &\quad + r\theta_c \mu^0 \theta_o \int_0^{t_1} D''(p(t)) e^{rt} \theta_o \frac{d\mu^0}{d\bar{Z}} e^{rt} dt \\ &= -\theta_c D'(p(t_1)) \lambda_o^0 r e^{rt_1} \frac{dt_1}{d\bar{Z}} + (\theta_o - \theta_c) D'(p(0)) \theta_o \frac{d\mu^0}{d\bar{Z}} \\ &\quad - \theta_c \frac{d\mu^0}{d\bar{Z}} \theta_o \int_0^{t_1} D''(p(t)) r e^{rt} \lambda_o^0 e^{rt} dt \end{aligned}$$

If  $D'' \leq 0$ ,  $-\theta_c \frac{d\mu^0}{d\bar{Z}} \theta_o \int_0^{t_1} D''(p(t)) r e^{rt} \lambda_o^0 e^{rt} dt < 0$  thus  $\frac{d^2 \lambda_o^0}{d\bar{Z} d\bar{Z}} \Big|_{\frac{d\lambda_o^0}{d\bar{Z}}=0} < 0$ . □



### 3.2 Coal then oil, $c_o > c_c$ .

If  $c_o > c_c$ , without carbon regulation, oil will not be used in that case. The carbon regulation may help oil to be used if its price becomes cheaper than coal price.

As shown in Table 3, four different cases exist. Both coal and oil are used to get to the ceiling, coal is used first, oil gets exhausted (Case 2.A). Both coal and oil are used to get to the ceiling, coal is used first, oil is not exhausted (Case 2.B). Only oil can be used to get to the ceiling (Case 2.D). Only coal is used to get to the ceiling (Case 2.C). The only relevant case to study is Case 2.A where both resources are used, and oil gets exhausted. We show the conditions over parameters to get this case below (Lemma 10).

When oil is used after coal and fully exhausted, the solution  $\{\lambda_o^0, \mu^0, t_1, \underline{t}\}$ <sup>5</sup> satisfies:

$$c_o + \lambda_o e^{rt_1} + \theta_o \mu^0 e^{rt_1} = c_c + \theta_c \mu^0 e^{rt_1} \quad (3.5)$$

$$c_o + \lambda_o e^{r\underline{t}} + \theta_o \mu^0 e^{r\underline{t}} = c_s \quad (3.6)$$

$$\int_{t_1}^{\underline{t}} D(p(t)) dt = X_o^0 \quad (3.7)$$

$$\int_0^{t_1} \theta_c D(p(t)) dt \theta_o + \theta_o X_o^0 = \bar{Z} - Z^0. \quad (3.8)$$

We assume that resource extraction is as described by Case 2.A and that after a marginal decrease of the carbon ceiling we still stay in that case. Case 2.A is described in Figure 4. Bold curves represent coal and oil prices and the medium curve represent the scarcity rent. The initial prices paths are described by the plain curves. Dotted curves represent these price and the scarcity after a decrease of the ceiling. Vertical bold Black lines represent the tax by unit of oil and coal and vertical bold Grey lines represent the oil profits at the date of switch from one resource to another. In that case, tightening the carbon regulation will increase the carbon tax, bring forward the date of switch to oil and the date the ceiling binds, and increase oil profits. Indeed, first let us remark that if the case is stable, oil consumption is unchanged, thus the price at which oil starts to be used is unchanged and

<sup>5</sup>Sets of equations describing the solution in the other cases are straightforward. For Case 2.A, the equation set is similar, except that equation 3.7 must be dropped and the scarcity rent is set to 0. For cases 2.D and 2.C, the solution is as described by the solution of the one-resource case, as shown above.

noted  $p_1$ . If the carbon tax is reduced, thus the date of switch must be postponed, and thus the global consumption of coal increases, that is not possible. Thus the carbon tax must increase (effect 1 in Figure 4) and the date of switch is brought forward (effect 2). The date the ceiling is reached is also brought forward (effect 4), and the length of the oil consumption period is unchanged since the sum of the scarcity rent and the carbon tax at the switch date is unchanged. Since the switch price is unchanged, the value of the tax at the date of switch is unchanged (vertical bold black lines), thus profits at that date must be unchanged (vertical bold Grey lines). Since the date of switch is brought forward, the initial scarcity rent must increase (effect 5) to keep the profit of time  $t_1$  unchanged.

**Lemma 10.** *If  $c_c < c_o$ , different cases can arise.*

- If  $c_s < \frac{\theta_c c_o - \theta_o c_c}{\theta_c - \theta_o}$ , only coal is used to get to the ceiling (Case 2.C) and oil is never used.
- If  $c_s > \frac{\theta_c c_o - \theta_o c_c}{\theta_c - \theta_o}$ , then oil is used when the ceiling is about to bind and  $\exists Z^*$  such that:
  1. If  $\bar{Z} < Z^*$  and  $X > \frac{\bar{Z} - Z_0}{\theta_0}$ , only oil is used to get to the ceiling and coal is never used (Case 2.D);
  2. If  $\bar{Z} > Z^*$  and  $X_o^0 > \frac{Z^* - Z_0}{\theta_0}$ , coal is used at the beginning, then oil is used to get to the ceiling but oil is not exhausted (Case 2.B) ;
  3. Otherwise, if  $X \leq \min(\frac{Z^* - Z_0}{\theta_0}, \frac{\bar{Z} - Z_0}{\theta_0})$  coal is used at the beginning, then oil is used to get to the ceiling and is exhausted (Case 2.A).

*Proof.* First, remark that if coal is used at the date of switch with solar (the date the ceiling is reached), then oil is never used. Indeed, assume that oil is used first and then coal is used. If coal is used at the switch date with solar  $\underline{t}$ , then it must be the case that  $c_o + (\theta_o \mu^0 + \lambda_o^0) e^{r\underline{t}} > c_c + \theta_c \mu^0 e^{r\underline{t}}$ , but then  $\forall t \leq \underline{t}$ ,  $c_o + (\theta_o \mu^0 + \lambda_o^0) e^{rt} > c_c + \theta_c \mu^0 e^{rt}$ , so that oil is never used. If coal is used alone until the date of switch with solar, then the carbon tax at this date,  $\mu^0 e^{r\underline{t}}$ , satisfies:

$$c_c + \theta_c \mu^0 e^{r\bar{t}} = c_s$$

A necessary condition for coal to be used alone until the ceiling is reached is that:

$$c_o + \theta_o \mu^0 e^{r\bar{t}} > c_c + \theta_c \mu^0 e^{r\bar{t}}$$

Using that  $c_c + \theta_c \mu^0 e^{r\bar{t}} = c_s$ , a necessary condition is that:

$$c_o - c_c \geq \frac{\theta_c - \theta_o}{\theta_o} (c_s - c_c)$$

which can be rewritten:

$$c_s \leq \frac{\theta_c c_o - \theta_o c_c}{\theta_c - \theta_o}$$

Similarly, it is easy to show that a necessary condition for oil to be used at the binding date is that  $c_s > \frac{\theta_c c_o - \theta_o c_c}{\theta_c - \theta_o}$ . So that oil is used at the date the ceiling is reached if and only if  $c_s \geq \frac{\theta_c c_o - \theta_o c_c}{\theta_c - \theta_o}$ .

Assume now that  $c_s > \frac{\theta_c c_o - \theta_o c_c}{\theta_c - \theta_o}$ , then oil is used at the binding date. Carbon tax  $\tau$  at the date oil starts to be used is such that:  $c_o + \lambda_o^0 e^{r\bar{t}} + \theta_o \tau \leq c_c + \theta_c \tau$ , implying that:

$$\tau \geq \frac{c_o - c_c}{\theta_c - \theta_o}$$

The lowest possible oil price path is  $p(t) = c_o + \frac{c_o - c_c}{\theta_c - \theta_o} e^{rt}$ . Call  $T^*$  the date such that:

$$c_o + \frac{c_o - c_c}{\theta_c - \theta_o} e^{rT^*} = c_s$$

Then the maximum amount of oil that can be consumed, if  $c_s > \frac{\theta_c c_o - \theta_o c_c}{\theta_c - \theta_o}$  is:

$$X^* = \int_0^{T^*} D(c_o + \frac{c_o - c_c}{\theta_c - \theta_o} e^{rt}) dt$$

If  $X_o^0 > X^*$ , oil is not exhausted. If  $X_o^0 > X^*$  and  $\bar{Z} > Z_0 + \theta_0 X^* \equiv Z^*$ , then coal is used first, then an amount  $X^*$  of oil is used to get to the ceiling, oil is not exhausted (Case 2.B). If  $X_o^0 < X^*$  and  $\bar{Z} < Z_0 + \theta_0 X$ , then only oil is used to get to the ceiling and oil is not exhausted (Case 2.D). If  $X_o^0 < X^*$  and  $\bar{Z} > Z_0 + \theta_0 X_o^0$ , then coal is used first, then oil is used to get to the ceiling and is exhausted (Case 2.A). The different cases when oil is exhausted are indicated in Figure 4.  $\square$

**Proposition 11.** *If  $c_s > \frac{\theta_o c_c - \theta_c c_o}{\theta_o - \theta_c}$ , and  $X_o^0 < \min(X^*, \frac{\bar{Z} - Z_0}{\theta_o})$ , then:*

$$\frac{d\lambda_o^0}{d\bar{Z}} < 0$$

*As long as both resources are used and oil is fully used, tightening the ceiling constraint increases the scarcity rent of oil.*

*Proof.* See supra.  $\square$

## 4 Conclusions

This paper studies how oil owners can benefit from carbon taxation even if tax revenues are not redistributed. Taxing carbon emissions have different effects on the profits of fossil fuels owners depending on the characteristics of the fossil fuels they own (extraction and delivery costs, reserves size and carbon content). We built a Hotelling-like model with three energy resources: oil (exhaustible, polluting), coal (non exhaustible, very polluting) and solar energy (non exhaustible, non polluting). The  $CO_2$  concentration must be kept under a carbon ceiling. Natural dilution is negligible. Optimal extraction path is decentralized by a tax on emissions, and tax revenues are not redistributed. Our results cast some light on redistributive effects of carbon taxation. Contrary to Liski & Tahvonen (2004), Bergstrom (1982), we show that capturing oil rents with Pigouvian taxation is not possible when pollution comes from various sources, and that taxing emissions may increase the profits of some fossil fuels owners. We first characterized the different extraction paths depending on

parameters and focused on cases where both oil and coal are used and oil gets exhausted. When oil is used before coal, if oil is scarce enough or if the extraction cost of oil is close enough to the extraction cost of coal, tightening the carbon regulation increases the profits of oil owners. Indeed, if the resource is relatively scarce, the switch from oil to coal occurs early. To compensate the increase of the price of coal, profits must increase early. If the oil extraction cost is close enough to the coal extraction cost, tightening the carbon regulation will lead to increase the comparative advantage of oil in terms of pollution. Indeed with similar extraction costs, coal and oil would be perfect substitutes if no carbon regulation was implemented. If oil is sufficiently less polluting than coal, oil owners will benefit from a tighter carbon regulation. If the elasticity of demand is low enough, tightening the carbon ceiling increases oil profits. If the energy demand is concave or linear, oil profits cannot exhibit a U-shape when the carbon ceiling is strengthened. If oil is used after coal (oil is more expensive to extract) and exhausted, tightening the carbon regulation reduces coal consumption, lets global oil consumption unchanged and increases the profits of oil owners.

Our results leads to reconsider the debate over compensations for losses induced by carbon taxation, claimed for instance by OPEC countries. Major coal exporters are likely to be durably not sensitive to pro-mitigation arguments as long as their losses are not at least partially compensated. Oil and gas exporters may be more easily convinced about the necessity of carbon regulation since they may take direct advantage of carbon taxation. Note that countries' benefits from mitigation also depend on their vulnerability to climate change. Gas and oil owners from Middle-East may be those who win the most in a carbon-regulated world.

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Figure 4.1: Oil profits, the carbon tax and the price paths when the carbon ceiling is lowered,  $c_o < c_c$

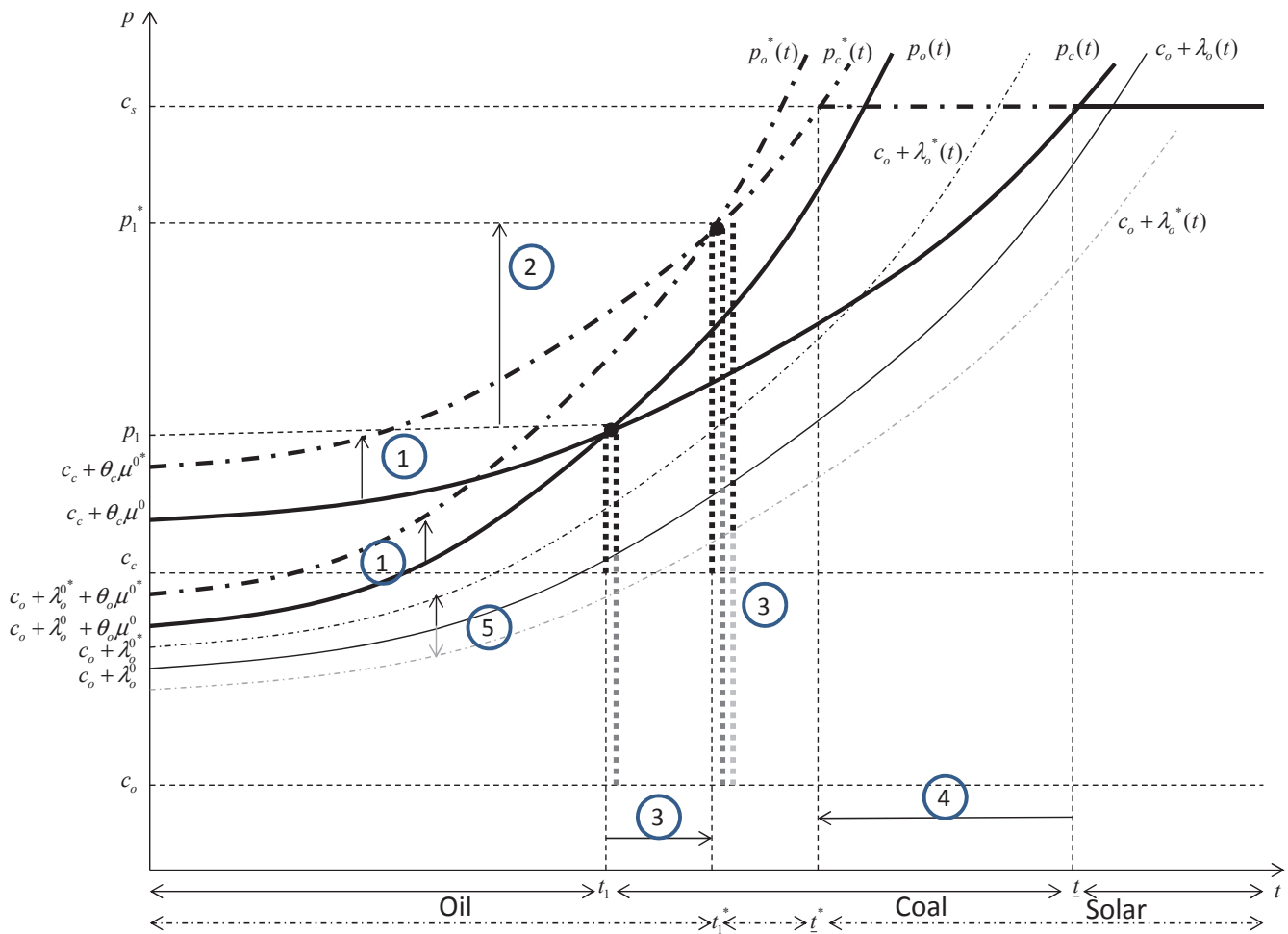




Figure 4.2: Characterization of the different cases when  $c_o > c_c$

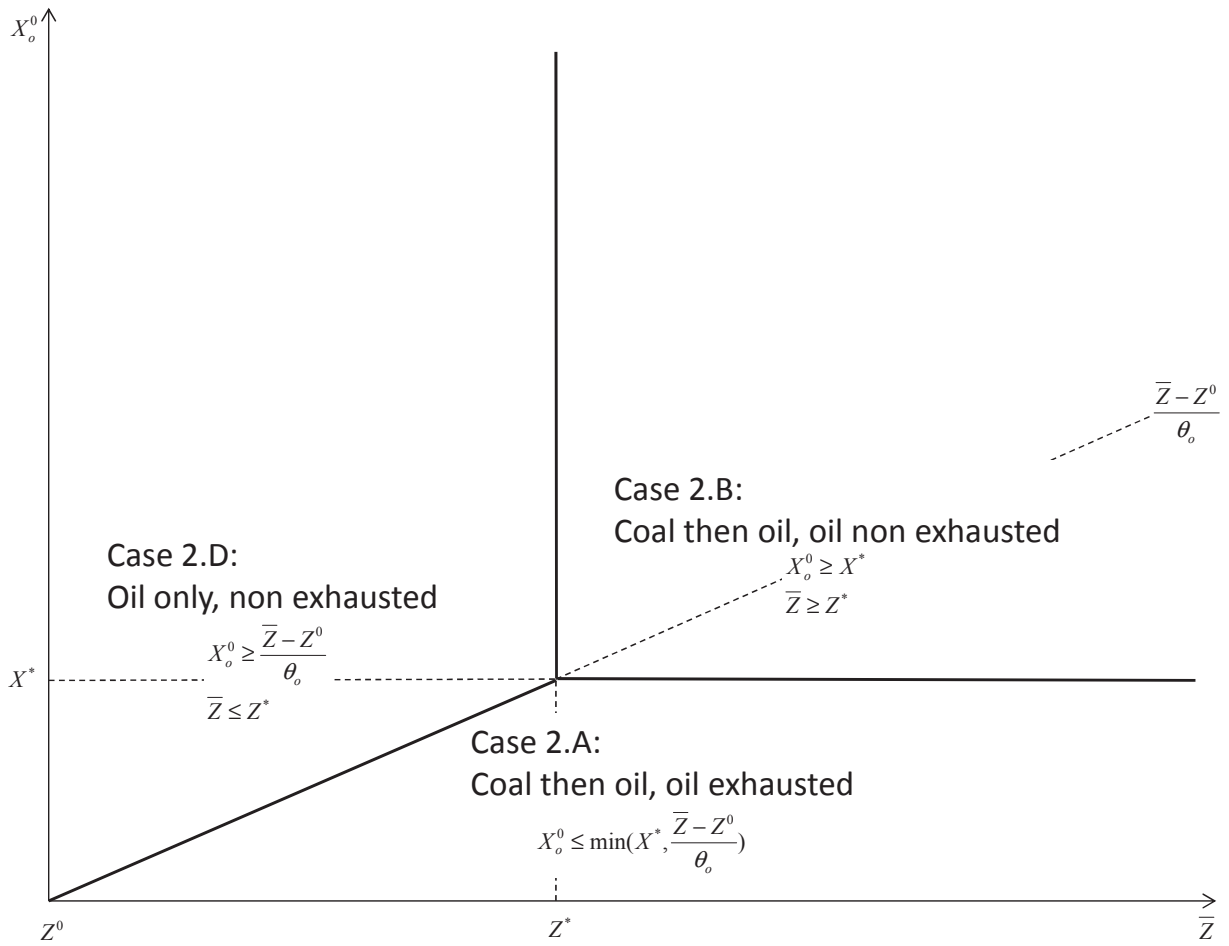


Figure 4.3: Oil profits, the carbon tax and the price paths when the carbon ceiling is lowered,  $c_o > c_c$

