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From Carbon Regulation**

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PARIS-JOURDAN SCIENCES ECONOMIQUES

48, Bd JOURDAN – E.N.S. – 75014 PARIS
TÉL. : 33(0) 1 43 13 63 00 – FAX : 33 (0) 1 43 13 63 10
www.pse.ens.fr

The Grey Paradox: How Owners Of Carbon-emitting Resources Can Benefit From Optimal Carbon Taxation.

Renaud Coulomb * Fanny Henriet †

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Abstract

We build a model of resource extraction to highlight how CO_2 taxation can increase the profits of owners of a carbon-emitting exhaustible resource. This resource competes with a dirtier abundant resource and a clean backstop. CO_2 concentration has to be kept under a ceiling. The optimum is decentralized by a carbon tax. As the carbon ceiling is tightened, the exhaustible-resource rent, and thus profits, is partly captured by the tax levier (the “capture effect”), but the dirtier resource is made less competitive (the “competition effect”). The role of resource endowments, pollution contents, extraction costs and demand elasticity is analyzed.

Keywords: Carbon Taxation, Fossil Fuels, Global Warming, Non-renewable Resources, OPEC.

JEL Classification: H21, H23, Q31, Q38, Q41, Q48, Q54, Q58.

*Ecole Centrale Paris, Grande Voie des Vignes, 92 295 Châtenay-Malabry Cedex, France, & London School of Economics and Political Science (LSE), Houghton Street, London WC2A 2AE, UK. Email: renaud.coulomb@gmail.com.

†Paris School of Economics-CNRS, Maison des Sciences Economiques, 106-112 Boulevard de l'Hôpital, 75013 Paris, France. Email: fanny.henriet@psemail.eu.

1 Introduction

Former OPEC Secretary General, Rilwanu Lukman, has declared¹ *“Especially vulnerable are the oil producing developing countries, which are mainly OPEC member countries, [...] their principal revenue-earner, petroleum, is inextricably associated with the downside of the negotiations. It is important to ensure that measures taken to combat climate change do not place an unfair burden on oil.”* In line with this position, OPEC has claimed for compensation in international regulations.² This paper analyzes the impact of the tightening of a carbon cap on CO_2 concentration on the profits of the owners of an exhaustible polluting resource when energy resources are optimally extracted and the optimum is decentralized by a carbon tax, without any compensation.

The effect of taxing carbon emissions on the profits of fossil-fuel owners depends on the characteristics of their fossil fuels (recoverable reserves, pollution content, and extraction cost). Contrary to the OPEC position, we find that the profits of owners of a polluting exhaustible resource, such as conventional oil or gas, may rise under (optimal) carbon taxation if a dirtier abundant resource is also used, even if the tax revenues are not redistributed (the Grey Paradox). Two characteristics of these resources lie behind this result: their exhaustibility and their relatively low pollution content as compared to abundant fossil fuels. First, they are likely to be exhausted unless carbon regulation is very stringent, so that their cumulative consumption to a certain extent does not depend on carbon regulation. Second, these resources are (or will be) in competition with more polluting and abundant fossil fuels, like coal or unconventional oil: see Table 1. The after-tax price of these not-too-polluting exhaustible resources may rise more than the carbon tax, leading to greater profits for resource owners.

Taxing carbon emissions will have varying effects on countries' wealth depending on their fossil-fuels reserves. Fossil-fuel reserves are distributed very

¹The 6th Conference of the Parties (COP) to the UN Framework Convention on Climate Change (UNFCCC) - The Hague, November 2000.

²See more recently the position defended by Abdalla Salem El-Badri, OPEC Secretary General at the COP15 to the UNFCCC - Copenhagen, December 2009.

unequally across countries, and, with the noticeable exception of the Commonwealth of Independent States (CIS), and more specifically Russia, countries with large endowments of conventional oil or gas have only little coal or unconventional oil, as shown in Table 2. OPEC-Gulf countries own 49.6% and 39.8% of world reserves of oil and natural gas, but only 0.1% of coal reserves and their reserves of unconventional oil are nil. If carbon taxation reduces the profits of coal or unconventional-oil owners but increases the profits of gas or conventional-oil owners, OPEC-Gulf countries may become richer as a result of carbon taxation, contrary to countries with large endowments of more polluting resources.

The distributional aspects of carbon regulation have received only little attention in the theoretical literature. This literature has focused on capturing rents from resource owners via the taxation of externalities (Bergstrom 1982, Wirl 1995, Rubio & Escriche 2001, Liski & Tahvonen 2004) or via tariffs (Brander & Djajic 1983, Karp 1984). Our paper shows that taxing CO_2 emissions may increase the profits of fossil-fuel owners who generate them, without there being any possibility that the regulator capture this extra rent.

The empirical literature has made a number of attempts to evaluate the impact of long-term carbon regulation or the Kyoto Protocol on fossil-fuel prices and oil and gas revenues. Most work based on simulations of energy-economic models has concluded that OPEC will suffer losses from the implementation of the Kyoto Protocol (Bernstein et al. 1999, Ghanem et al. 1999, McKibbin et al. 1999, Bartsch & Müller 2000, Polidano et al. 2000). The largest figure for the effect of the Kyoto Protocol on OPEC oil revenues is a 13% fall compared to the non-Kyoto scenario in 2010 (McKibbin et al. 1999). Reviewing this literature, Barnett et al. (2004) suggest three elements that may reduce losses for OPEC countries, but do not consider the role they play in detail: carbon leakage driven by the non-universality of the Kyoto Protocol, substitution of oil and gas for more polluting fossil fuels such as coal, and the limited availability of oil and gas in the future.³ A noticeable exception in this literature is

³Barnett et al. (2004) also argue that additional policies and measures such as reducing coal subsidies, measures to discourage the development of fossil-fuel production in developed

Persson et al. (2007). Carrying out simulations based on an energy-economic optimization model, they find that conventional-oil profits actually rise due to carbon regulation. However, none of these articles has produced analytical results regarding this question.

Our paper examines the impact of *optimal* carbon taxation on the profits of fossil-fuel owners. We thus first determine optimal resource extraction and the subsequent optimal carbon tax. This question has received considerable attention in the literature in a variety of settings: with a carbon ceiling on the CO_2 stock and one fossil fuel (Chakravorty et al. 2006) or several (Chakravorty et al. 2008), or with a continuous increasing damage function with one fossil fuel (Ulph & Ulph 1994, Tahvonen 1997) or several (van der Ploeg & Withagen 2012). However, none of these papers has analyzed how optimal carbon taxes affect fossil-fuel profits.

Following Chakravorty et al. (2006), we construct a Hotelling-like model where the CO_2 concentration must be kept under a carbon ceiling. This threshold can be considered as an exogenous constraint, for instance stemming from a Kyoto-like Protocol, or as the first-best carbon policy if damage can be approximated by a binary function with no marginal damage when the CO_2 concentration is below the threshold and infinite otherwise. The social planner seeks to maximize the total surplus, i.e. the sum of the consumer and producer surplus, taking account of the scarcity constraint and the carbon-cap constraint. As in van der Ploeg & Withagen (2012), consumer utility comes from three perfect-substitute energy sources: an exhaustible polluting resource, an abundant dirtier resource (the dirty backstop) and an abundant clean resource (the clean backstop). Each resource is distinguished by its carbon content, extraction cost and reserves. Since scarce polluting resources can be more expensive or cheaper to extract than abundant dirtier resources in real life, we explore both cases. As in Chakravorty et al. (2006, 2008) and van der Ploeg & Withagen (2012), fossil-fuel owners are in perfect competition;⁴ prof-

countries, the abandon of nuclear power, the diversification of OPEC economies, and the development of carbon sinks may limit the losses of OPEC countries.

⁴A number of pieces of empirical work (Ezzati 1976, MacAvoy 1982, Verleger 1982) explain oil prices changes using a competitive model.

its are thus only driven by resource scarcity. To implement optimal policy, the social planner can impose a worldwide carbon tax but cannot prevent the use of a particular resource or set a specific tax on each resource.

The Grey Paradox can be expressed as follows: the profits of owners of not-too-polluting exhaustible resources may rise due to (optimal) carbon taxation. We first show that a unique carbon tax path allows the optimum to be decentralized when the exhaustible resource is exhausted and the dirty backstop is used. This tax must equal the shadow cost of pollution, and marginal profit is equal to the scarcity rent. In the main part of the analysis, we consider how the scarcity rent changes as the carbon ceiling falls, when both polluting resources are extracted and the exhaustible resource is exhausted. The overall effect of a fall in the ceiling on the scarcity rent, and thus on profits, is shown to be ambiguous. On the one hand, there is a positive “competition effect” on the profits of the exhaustible-resource owners: their not-too-polluting exhaustible resource is (or will be) in competition with an even more polluting resource, which will be subject to a higher tax, this tends to increase their profits. On the other hand, there is a negative “capture effect” on the profits of the exhaustible-resource owners: their resource is subject to an increased carbon tax, decreasing the demand at each date and thus tending to lower their profits. If the exhaustible resource is cheaper to extract than the dirtier resource, tightening carbon regulation increases the profits of exhaustible-resource owners if any of the following hold: *(i)* its demand elasticity is low enough; *(ii)* its extraction cost is close enough to that of the dirty backstop; *(iii)* its pollution content is low enough (compared to that of the dirty backstop); or *(iv)* its initial stock is small enough. When the exhaustible resource is more expensive to extract than the dirty backstop, tightening carbon regulation increases the profits of exhaustible-resource owners. Introducing a technology that allows for capturing CO_2 at constant marginal cost does not change these results.

The remainder of this paper is organized as follows. Section 2 presents the main model. Section 3 then characterizes the optimal extraction path and the decentralization rule. Section 4 shows the effects of carbon regulation on the profits of exhaustible-resource owners, and Section 5 discusses some possible

extensions. Last, Section 6 concludes.

Fossil fuel	Reserves (EJ)	Resources (EJ)	Pollution content (gCO ₂ e/MJ)
Oil	9 032	19 495	
<i>Conv.</i>	7014	6 637	92
<i>Unconv.</i>	2018	12 858	106
Natural gas	7 415	29 842	
<i>Conv.</i>	7 240	11 671	76
<i>Unconv.</i>	175	18 171	72
Coal	21 952	473 893	110

Data for reserves and resources from BGR (2012). “Reserves” are proven volumes economically exploitable at today’s prices and using today’s technology. “Resources” are proven amounts which cannot currently be exploited for technical and/or economic reasons, as well as unproven but geologically possible energy resources which may be exploitable in future. Pollution contents are life-cycle greenhouse gas contents from Burnham et al. (2012) (equivalence between CO₂ and other greenhouse gases measured over 100 years). EJ = exajoules; MJ = megajoules. “Conv.” and “Unconv.” stand for “Conventional” and “Unconventional”.

Table 1: World reserves, resources and pollution contents of fossil fuels in 2011

2 The model

2.1 Assumptions and notation

We consider that utility is derived from energy consumption. Three different energy resources, which are perfect substitutes in demand, are available: an exhaustible resource, which is polluting, in quantity X_e^0 , an even more polluting non-exhaustible resource, and a non-exhaustible clean resource. The labels e, d, b respectively stand for the “exhaustible resource”, “dirty backstop” and “clean backstop”. The resources are labeled R_e, R_d and R_b . The extraction flow of resource i , $i = \{e, d, b\}$, is $x_i(t)$. The current flow of utility is thus $u(x_e(t) + x_d(t) + x_b(t))$. The decreasing energy demand function is $D(\cdot)$, and p_i , $i = \{e, d, b\}$, are the resource prices. We define θ_i as the pollution content of resource i : the use of one unit of resource i leads to θ_i units of CO₂. We assume that $0 < \theta_e < \theta_d$ and that $\theta_b = 0$. The extraction cost of resource i is

	Oil			Natural gas			Coal
	Total	Conv.	Unconv.	Total	Conv.	Unconv.	
Europe	1.0	1.3	0.0	2.2	2.3	0.0	8.6
CIS	8.1	10.4	0.0	31.9	32.7	1.0	20.7
Africa	8.3	10.7	0.0	7.5	7.7	0.0	3.5
Middle East	50.2	64.7	0.0	40.8	41.8	0.0	0.1
Austral-Asia	2.6	3.4	0.0	8.6	8.2	27.0	40.3
North America	15.5	3.8	56.1	5.0	3.4	72.0	25.4
Latin America	14.3	5.8	43.9	3.9	4.0	0.0	1.4
World	100	100	100	100	100	100	100
OPEC 2009	69.3	76.6	43.9	48.5	49.7	0.0	0.2
OPEC-Gulf	49.6	63.8	0.0	39.8	40.7	0.0	0.1
OECD 2000	16.7	5.3	56.1	9.1	7.1	92.3	43.2

Calculus using data from BGR (2012).

Table 2: Share of world fossil-fuel reserves by world region and economic policy organization in 2011

c_i : we assume that $0 < c_e < c_b$ and that $0 < c_d < c_b$. The dirty backstop is non-exhaustible (or, equivalently, the dirtier resource is abundant enough not to be exhausted for the ceiling regulation we choose). The clean resource is available in infinite quantity at cost c_b . The initial amount of R_e is written X_e^0 , and the change in its current stock is:

$$\dot{X}_e(t) = -x_e(t).$$

We assume that the social discount rate, r , is constant, this is also the interest rate. The carbon stock, $Z(t)$, has to be kept under a threshold \bar{Z} . This threshold can be considered as an exogenous constraint, for instance stemming from a Kyoto-like Protocol. This type of regulation is closer to first-best carbon regulation than a constant tax policy, as the marginal damage increases steeply with the atmospheric carbon stock.⁵ Since the dirty backstop is available in infinite quantity, the ceiling is binding for any value of \bar{Z} . We assume that

⁵All of the results below hold with a constant carbon tax.

$\bar{Z} > Z^0$. There is no natural decay of carbon, as in van der Ploeg & Withagen (2012).⁶ The change in the carbon stock over time is simply given by:

$$\dot{Z}(t) = \theta_e x_e(t) + \theta_d x_d(t).$$

The social planner has soft power to implement optimal policy: he can impose a tax on CO_2 emissions but cannot prevent the use of a particular resource or set a specific tax or quota for each different resource. This tax can be paid by consumers (demand side) or by fossil-fuel providers (supply side). The tax is worldwide: no market can be exempt of the tax. There is no redistribution of tax revenues to fossil-fuel owners. Markets are competitive, thus profits come only from the scarcity of the exhaustible resource.

2.2 The welfare-maximization program

The social planner wishes to establish the extraction $\{x_e(t), x_d(t), x_b(t)\}$ which maximizes the net discounted social surplus⁷ under the environmental constraint:

$$\int_0^\infty e^{-rt} \left(u(x_e(t) + x_d(t) + x_b(t)) - c_e x_e(t) - c_d x_d(t) - c_b x_b(t) \right) dt$$

s.t. $i = \{e, d, b\}$,

$$\dot{X}_e(t) = -x_e(t)$$

$$\dot{Z}(t) = \theta_e x_e(t) + \theta_d x_d(t)$$

$$Z(t) \leq \bar{Z}$$

$$X_e(t), x_i(t) \geq 0$$

with Z^0 and X_e^0 given.

⁶No consensus exists over the form of natural dilution; it is, however, acknowledged that it is relatively small. According to Allen et al. (2009), the relationship between cumulative emissions and global warming is insensitive to the emission pathway. We thus assume that natural dilution is negligible.

⁷The total surplus is the sum of the consumer surplus and the producer one.

Let $\lambda_e(t)$ be the shadow value of the remaining stock of the exhaustible resource $X_e(t)$ and $\mu(t)$ that of the pollution stock $Z(t)$. The transversality conditions are given by:

$$\lim_{t \rightarrow \infty} \lambda_e(t) e^{-rt} X_e(t) = 0 \quad (2.1)$$

$$\lim_{t \rightarrow \infty} \mu(t) e^{-rt} Z(t) = 0. \quad (2.2)$$

Equation 2.1 simply states that the exhaustible resource must be exhausted in the long run if the scarcity rent is positive.

2.3 The first-order conditions

We define the current-value Hamiltonian as:

$$\begin{aligned} H(t) = & u(x_e(t) + x_d(t) + x_b(t)) - c_e x_e(t) - c_d x_d(t) - c_b x_b(t) \\ & - \lambda_e(t) x_e(t) - \mu(t) (\theta_e x_e(t) + \theta_d x_d(t)). \end{aligned}$$

This has the following slackness conditions:

$$\begin{aligned} \nu(t) &\geq 0 \quad \text{and} \quad \nu(t)(\bar{Z} - Z(t)) = 0 \\ \beta(t) &\geq 0 \quad \text{and} \quad \beta(t)X_e(t) = 0 \\ \gamma(t) &\geq 0 \quad \text{and} \quad \gamma(t)x_e(t) = 0. \end{aligned}$$

For any control $\{x_e(t), x_d(t), x_b\}$ there exist co-state variables $\lambda_e(t)$ and $\mu(t)$, that must satisfy the following conditions, as well as the transversality and slackness conditions:

$$\dot{\lambda}_e(t) = r\lambda_e(t) - \frac{\partial H(t)}{\partial X_e(t)} \iff \dot{\lambda}_e(t) = r\lambda_e(t) + \beta(t) \quad (2.3)$$

$$\dot{\mu}(t) = r\mu(t) - \frac{\partial H(t)}{\partial Z(t)} \iff \dot{\mu}(t) = r\mu(t) + \nu(t) \quad (2.4)$$

$$\frac{\partial H(t)}{\partial x_e(t)} = 0 \iff p_e(t) = c_e + \lambda_e(t) + \theta_e \mu(t) \quad (2.5)$$

$$\frac{\partial H(t)}{\partial x_d(t)} = 0 \iff p_d(t) = c_d + \theta_d \mu(t) \quad (2.6)$$

$$\frac{\partial H(t)}{\partial x_b(t)} = 0 \iff p_b(t) = c_b. \quad (2.7)$$

The co-state variable $\lambda_e(t)$ represents the current value of the scarcity rent of the exhaustible resource. As shown in Hotelling (1931), this increases at a rate of r : the discounted net marginal surplus of extraction must be constant. Along the optimal path, extracting a supplementary unit must be equivalent to saving it for later use. Writing $\lambda_e^0 \equiv \lambda_e(0)$, it can be shown that:

$$\lambda_e(t) = \lambda_e^0 e^{rt}.$$

The co-state variable $\mu(t)$ represents the current value of the shadow cost of marginal pollution. This exhibits a familiar pattern driven by the ceiling form of the carbon regulation. If the ceiling does not bind (but will bind), the pollution cost rises at the rate of the discount rate. The intuition behind this result is similar to that in the Hotelling rule, as emitting CO_2 can be seen as extracting clean air from a reservoir with an initial stock of clean air defined by $\bar{Z} - Z^0$. Writing $\mu^0 \equiv \mu(0)$, we have:

$$\mu(t) = \mu^0 e^{rt}.$$

We show below that when both polluting resources are used and R_e becomes exhausted, a unique carbon tax exists that allows the equilibrium to be decentralized (Lemma 3). This tax equals the shadow cost of pollution, $\mu(t)$, and the current marginal profit equals the scarcity rent, $\lambda_e(t)$.

The optimal price of R_e is simply the sum of the extraction cost, pollution cost and scarcity rent from equation 2.5. The optimal price of R_d is the sum of the extraction cost and pollution cost from equation 2.6. The unitary pollution cost of carbon is independent of the emission source, but pollution costs per unit of energy differ due to the variety of pollution contents.

3 The optimal extraction path

3.1 Ordering resource extraction

The different extraction paths are described in Table 3. Only the cases where both resources are used and R_e becomes exhausted (Cases A1 and B1) are relevant for the analysis of the Grey Paradox: we focus on these two cases. However, we fully characterize the different extraction paths in order to determine the parameter conditions which yield the relevant cases.

	Extraction costs	The exhaustible resource	The dirty backstop
Case A1	$c_d > c_e$	Used, exh.	Used
Case A2	...	Used, not exh.*	Not used
Case B1	$c_d < c_e$	Used, exh.	Used
Case B2	...	Used, not exh.	Used
Case B3	...	Not used	Used
Case B4	...	Used, not exh.*	Not used

“exh.” stands for “exhausted”. *If $X_e^0 = \frac{\bar{Z}-Z^0}{\theta_e}$, there is a borderline case: only R_e is used and becomes exhausted.

Table 3: The different extraction paths

Let remark the following properties of the extraction and price paths. First, the energy price is continuous over time and equals the minimum of resources prices. Second there is no stop-and-go in the use of any resource and no simultaneous use. Resource i price ($i = \{e, d, b\}$) is of the form: $c_i + d_i e^{rt}$, $d_i \in \mathbb{R}^+$, so that price paths can at most cross once, given the different c_i s. Finally, without natural absorption and carbon sequestration, the maximum amount of pollution put into the atmosphere is fixed and equals $\bar{Z} - Z^0$. We will stop using fossil fuels once the CO_2 concentration reaches the ceiling, thus the date at which the ceiling binds corresponds to the date of the switch to the clean backstop.

Recall that R_e is less polluting than R_d ($\theta_e < \theta_d$) in the whole paper. Two general cases appear: R_e is cheaper or more expensive to extract than R_d ($c_e < c_d$ or $c_e > c_d$).

3.1.1 The exhaustible resource is cheaper to extract than the dirty backstop

If $c_e < c_d$, the exhaustible resource R_e is necessarily used first. This result conforms to the Herfindahl principle (Herfindahl 1967).⁸ As shown in Table 3, two cases can occur depending on whether both polluting resources are used to get to the ceiling (R_e is exhausted, Case A1) or only R_e is used (so that R_d is never used, Case A2). Lemma 1 immediately follows:

Lemma 1. *If the exhaustible resource is cheaper to extract than the dirty backstop, $c_e < c_d$, the following cases arise:*

- *If $X_e^0 < \frac{\bar{Z}-Z^0}{\theta_e}$, both polluting resources are used, and the exhaustible resource is exhausted (Case A1);*
- *If $X_e^0 \geq \frac{\bar{Z}-Z^0}{\theta_e}$, only the exhaustible resource is used to get to the ceiling (Case A2).*

The relevant case for analysis here is Case A1, where both polluting resources are used, and the exhaustible resource is exhausted. In this case, the extraction path is composed of three phases, as represented on Figure 1:

- Phase 1, $[0, t^s]$: the exhaustible resource is used, the energy price equals the price of this resource, $p(t) = p_e(t) = c_e + \lambda_e(t) + \theta_e \mu(t)$, and the atmospheric carbon stock is increasing but is strictly below the ceiling value. This resource becomes exhausted at time t^s , when its price equals the price of the dirty backstop: $p(t^s) = p_e(t^s) = p_d(t^s)$.
- Phase 2, $]t^s, \underline{t}]$: the dirty backstop is used, the energy price equals the price of this resource, $p(t) = p_d(t) = c_d + \theta_d \mu(t)$, and the atmospheric carbon stock is increasing. At date \underline{t} , the dirty-backstop price equals the clean-backstop price, $p(\underline{t}) = p_d(\underline{t}) = p_b(\underline{t}) = c_b$, and the carbon stock reaches the ceiling value, $Z(\underline{t}) = \bar{Z}$.

⁸Assume, by way of contradiction, that R_d is used first. Recall that the scarcity rent and the pollution cost increase at rate r . If $c_e < c_d$ and $c_e + \lambda_e^0 + \theta_e \mu^0 > c_d + \theta_d \mu^0$, which is the case if R_d is used first, then it is easy to see that for all t , $p_d(t) < p_e(t)$: only R_d is used until the ceiling is reached. But then, lowering λ_e^0 and increasing μ^0 , so that R_e is used first, would be preferred by the social planner, as R_e is cheaper to extract and less polluting.

- Phase 3, $]\underline{t}, \infty[$: only the clean backstop is used and the energy price is constant and equals its price: $p(t) = p_b(t) = c_b$.

[Insert Figure 1 here.]

Finding the optimal energy price requires us to determine the initial scarcity rent, λ_e^0 , the initial shadow cost of pollution, μ^0 , the date of the switch from R_e to R_d , t^s , and the date the ceiling is reached, \underline{t} . The solution $(\lambda_e^0, \mu^0, t^s, \underline{t})$, when extraction is of Case A1 type,⁹ must satisfy the following conditions:

$$c_e + \lambda_e^0 e^{rt^s} + \theta_e \mu^0 e^{rt^s} = c_d + \theta_d \mu^0 e^{rt^s} \quad (3.1)$$

$$c_d + \theta_d \mu^0 e^{r\underline{t}} = c_b \quad (3.2)$$

$$\int_0^{t^s} D(c_e + \lambda_e^0 e^{rt} + \theta_e \mu^0 e^{rt}) dt = X_e^0 \quad (3.3)$$

$$\theta_e X_e^0 + \int_{t^s}^{\underline{t}} \theta_d D(c_d + \theta_d \mu^0 e^{rt}) dt = \bar{Z} - Z^0. \quad (3.4)$$

Equations 3.1 and 3.2 express the continuity of the energy price between phases. Equation 3.3 reflects R_e exhaustion at date t^s and equation 3.4 shows that the environmental constraint binds at time \underline{t} .

Remark that if $X_e^0 = \frac{\bar{Z} - Z^0}{\theta_e}$, only the exhaustible resource is used and it become exhausted (borderline case between A1 and A2). In this case, the scarcity rent and the carbon tax are undefined, and only the sum of the carbon tax and the rent is defined.

3.1.2 The exhaustible resource is more expensive to extract than the dirty backstop

If $c_e > c_d$, without carbon regulation the exhaustible resource is never used.¹⁰ Carbon regulation may increase the price of the dirty backstop more than that of the exhaustible resource. The exhaustible resource may be used if its after-tax price drops below that of the dirty backstop.

⁹For Case A2, the solution is as described by the solution of the one-resource case.

¹⁰Note that were R_d to be less polluting than R_e and $c_d < c_e$, R_e would never be used with or without carbon regulation.

As shown in Table 3, four different cases pertain. First, both the dirty backstop and the exhaustible resource are used to reach the ceiling, the dirty backstop is used first, and the exhaustible resource becomes exhausted (Case *B1*). Second, both polluting resources are used to reach the ceiling, the dirty backstop first, and the exhaustible resource is not exhausted (Case *B2*). Third, only the dirty backstop is used to reach the ceiling (Case *B3*). Last, only the exhaustible resource is used to reach the ceiling (Case *B4*). The relevant case for analysis here is Case *B1*, where both resources are used, and the exhaustible resource is exhausted. We show the parameter conditions for this case to come about below in Lemma 2. The different cases when R_e is used are indicated in Figure 2.

Lemma 2. *If the dirty backstop is cheaper to extract than the exhaustible resource, $c_d < c_e$, the following cases arise:*

- *If $c_b \leq \frac{\theta_d c_e - \theta_e c_d}{\theta_d - \theta_e}$, only the dirty backstop is used to reach the ceiling (Case *B3*).*
- *If $c_b > \frac{\theta_d c_e - \theta_e c_d}{\theta_d - \theta_e}$, then the exhaustible resource is used when the ceiling is about to bind and there exists Z^* such that:*
 1. *If $\bar{Z} \leq Z^*$ and $X_e^0 \geq \frac{\bar{Z} - Z^0}{\theta_e}$, only the exhaustible resource is used to reach the ceiling (Case *B4*);*
 2. *If $\bar{Z} > Z^*$ and $X_e^0 > \frac{Z^* - Z^0}{\theta_e}$, the dirty backstop is used, and the exhaustible resource is then used to reach the ceiling but not exhausted (Case *B2*);*
 3. *Otherwise, if $X_e^0 < \frac{\bar{Z} - Z^0}{\theta_e}$ and $X_e^0 \leq \frac{Z^* - Z^0}{\theta_e}$ the dirty backstop is used at the beginning, and the exhaustible resource is then used to reach the ceiling and is exhausted (Case *B1*).*

Proof appears in Appendix A.1.

[Insert Figure 2 here.]

The relevant case for analysis here is Case *B1*, where both polluting resources are used, and the exhaustible resource is exhausted. In this case, the extraction path is composed of three phases, as represented on Figure 3.:

- Phase 1, $[0, t^s]$: the dirty backstop is used, the energy price equals the dirty backstop price, $p(t) = p_d(t) = c_d + \theta_d \mu(t)$, and the atmospheric carbon stock is increasing but is strictly below the ceiling. At date t^s , this price equals the price of the exhaustible resource, $p(t^s) = p_d(t^s) = p_e(t^s)$.
- Phase 2, $]t^s, \underline{t}]$: the exhaustible resource is used, the energy price equals the price of this resource, $p(t) = p_e(t) = c_e + \lambda_e(t) + \theta_e \mu(t)$, and the atmospheric carbon stock is increasing. This resource becomes exhausted at time \underline{t} , when its price equals the clean-backstop price, $p(\underline{t}) = p_e(\underline{t}) = p_b(\underline{t}) = c_b$, and the carbon stock reaches the ceiling value, $Z(\underline{t}) = \bar{Z}$.
- Phase 3, $]\underline{t}, \infty[$: only the clean backstop is used and the energy price is constant and equals the clean backstop price, $p(t) = p_b(t) = c_b$.

[Insert Figure 3 here.]

Finding the optimal energy price requires us to determine the initial scarcity rent, λ_e^0 , the initial shadow cost of pollution, μ^0 , the date of the switch from R_d to R_e , t^s , and the date the ceiling is reached, \underline{t} . When extraction is of Case *B1* type,¹¹ the exhaustible resource is used after the dirty backstop and exhausted, the solution $(\lambda_e^0, \mu^0, t^s, \underline{t})$ satisfies:

$$c_e + \lambda_e e^{rt^s} + \theta_e \mu^0 e^{rt^s} = c_d + \theta_d \mu^0 e^{rt^s} \quad (3.5)$$

$$c_e + \lambda_e e^{r\underline{t}} + \theta_e \mu^0 e^{r\underline{t}} = c_b \quad (3.6)$$

$$\int_{t^s}^{\underline{t}} D(p(t)) dt = X_e^0 \quad (3.7)$$

$$\int_0^{t^s} \theta_d D(p(t)) dt + \theta_e X_e^0 = \bar{Z} - Z^0. \quad (3.8)$$

¹¹For Case *B2*, the equation set is similar, except that equation 3.7 is dropped and the scarcity rent is set to 0. For cases *B3* and *B4*, the solution is as described by the solution of the one-resource case.

Equations 3.5 and 3.6 express the continuity of the energy price between phases. Equation 3.7 reflects R_e exhaustion at date \underline{t} and equation 3.8 shows that the environmental constraint binds at time \underline{t} .

Remark that if $c_b > \frac{\theta_e c_d - \theta_d c_e}{\theta_e - \theta_d}$ and $X_e^0 = \min(X^*, \frac{\bar{Z} - Z^0}{\theta_e})$, only the exhaustible resource is used and it becomes exhausted (borderline case between $B1$ and either $B2$ or $B4$). In this case, the scarcity rent and the carbon tax are undefined, and only the sum of the carbon tax and the rent is defined.

3.2 Decentralization via a tax on CO_2 emissions

The optimum is decentralized by a tax on CO_2 emissions. We assume that the tax scheme of the social planner is credible for individuals with perfect foresight, and that fossil-fuel owners are in perfect competition. Thus, profits come only from the scarcity of the exhaustible resource. We assume that fossil-fuel owners are faced with no threats regarding their property rights over their fossil-fuel reserves (see Strand 2010 on this issue). The carbon tax can be paid by consumers (demand side) or by fossil-fuel owners (extraction side). The tax is worldwide: no market can be exempt of the tax.

Lemma 3. *If both polluting resources are used and R_e becomes exhausted, the only way to decentralize the optimum is to set the carbon tax at the value of the shadow cost of pollution, $\mu(t)$.*

In a decentralized economy, with $\pi(t)$ being the profits per unit of the owners of R_e and $\tau(t)$ the carbon tax, optimal prices are:

$$\begin{aligned} p_e(t) &= c_e + \lambda_e(t) + \theta_e \mu(t) = c_e + \pi_e(t) + \theta_e \tau(t) \quad \text{when } p_e(t) < p_d(t) \\ p_d(t) &= c_d + \theta_d \mu(t) = c_d + \theta_d \tau(t) \quad \text{when } p_e(t) > p_d(t) \end{aligned}$$

where $\lambda_e(t)$ and $\mu(t)$ are defined by the set of necessary conditions over the continuity of the energy price, the exhaustion of R_e , and cumulative emissions (systems of equations 3.1-3.4 and 3.5-3.8). The optimal after-tax resource price paths are fully determined and unique.

Note that a necessary condition for the decentralization of the optimum is that profits increase at the interest rate, otherwise fossil-fuel owners would have an incentive to reallocate resource extraction over time in order to increase their profits. Resource prices net of extraction costs increase at the rate of the social discount rate, and profits must increase at this rate, so that the carbon tax must also increase at the social discount rate when the exhaustible resource or the dirty backstop is used. When the dirty backstop is used, from equation 2.6 the carbon tax equals $\mu(t)$. If the tax is different from $\mu(t)$ when R_e is used, the carbon tax must be discontinuous at the date of the switch from one fossil fuel to another. However, there is no downward jump in the carbon tax at this date, since otherwise the owners of the resource used in first position would have an incentive to postpone their extraction to increase their profits, and similarly there is no upward jump since the owners of the resource used in second position could increase their profits by bringing forward their extraction. It follows that the tax is continuous, and thus there is a unique initial value of the tax that allows the optimum to be decentralized, μ^0 .

Corollary 4. *When both polluting resources are used and R_e is exhausted, the current marginal profit equals $\lambda_e(t)$. Cumulative discounted profits, Π , are proportional to the initial scarcity rent, λ_e^0 and are given by:*

$$\Pi = \int_0^{\infty} e^{-rt} D(p(t)) \lambda_e(t) dt = \lambda_e^0 X_e^0.$$

Profits do not reflect the market power of resource owners but rather come from the scarcity of the resource. If R_e is not exhausted (cases A2, B2 and B4), perfect competition wipes out the profits of owners of R_e .

Recall that when only one polluting resource is used and exhausted, even if the energy price is well-defined over time, there are an infinite number of ways of setting the carbon tax and the scarcity rent to implement optimal extraction (Chakravorty et al. 2006). In this case, the tax can be set equal to the optimal price net of extraction costs to fully capture profits. In the present paper, profits could also be fully captured if only the exhaustible resource was used and if total emissions from burning the entire stock of this resource were

exactly equal to the initial clean air reservoir, $\bar{Z} - Z^0$. As we have just shown, this is no longer the case when two resources are used and when only pollution can be taxed.

We assume hereafter that there is no redistribution of tax revenues to fossil-fuel owners. Redistribution is not modeled *per se*. However, one can assume that tax revenues are redistributed to consumers whose utility is linear in money. Were tax revenues to be entirely redistributed to resource owners, the exhaustible-resource owners would see their wealth increase if their resource was still exhausted. This is due to the fact that the resource price must increase, so the sum of the profits and the tax must increase at each date.

4 The impact of carbon regulation on profits

4.1 The exhaustible resource is cheaper to extract than the dirty backstop

In this subsection we look at the impact of carbon regulation on the profits of exhaustible-resource owners when $c_e < c_d$. We consider Case A1, where both polluting resources are used to reach the ceiling ($X_e^0 < \frac{\bar{Z} - Z^0}{\theta_e}$).

[Insert Figure 5 here.]

Case A1 is described in Figure 5. The top (bottom) panel represents the case where the rent decreases (increases) after a fall of the ceiling i.e no Grey Paradox occurs (the Grey Paradox occurs). The bold curves represent R_d and R_e prices. The medium-thick curve shows the scarcity rent. Segments AB represent the tax by unit of R_e at the time of the switch from R_e to R_d . The dotted curves represent the new prices and the new scarcity rent after a fall in the ceiling. The symbol “*” indicates the new values after a fall in the ceiling.

Reducing the carbon ceiling increases the carbon tax. Let p^s be the switch price from R_e to R_d . First note that this switch price is such that the quantity of pollution generated by the use of R_d is equal to the value of the initial clean air reservoir $\bar{Z} - Z^0$ minus the stock of pollution generated by the burning of

the entire stock of R_e , $\theta_e X_e^0$. As the ceiling falls, R_e is totally exhausted but the quantity of R_d used to reach the ceiling must fall. R_d price¹² must rise, so that p^s must increase. It immediately follows that the date the ceiling binds is brought forward. The new price paths for R_e and R_d are represented by the bold dotted line.

If these new price paths were only due to an increase of the tax on R_d , and if the tax on R_e was unchanged at the switch date ($AB = A^*C^*$), then R_e before-tax price would be equal to $c_e + \lambda_e^{comp}(t)$, represented by the top medium-thick dotted line. The initial scarcity rent would increase from λ_e^0 to λ_e^{comp} , with $\lambda_e^{comp} \equiv \lambda_e^{comp}(0)$. We call it the “competition effect” since it results from an increase of R_d price that makes it less competitive. But this effect is opposed to a “capture effect”: as the tax increase also applies to R_e , a part of the rent, λ_e^{comp} , is captured. Its resulting before-tax price path, $c_e + \lambda_e^*(t)$, passes through B^* instead of C^* at the switch date: $c_e + \lambda_e^*(t^{s*}) = c_e + \lambda_e^{comp}(t^{s*}) - \theta_e d\mu^s$. The two effects are linked as $\theta_e d\mu^s = (\theta_e/\theta_d) dp^s$. Given the pollution differential, the increase in the switch price is larger than the increase of the tax on R_e , it follows that the current scarcity rent, at the switch date, rises after the carbon ceiling tightens (B^* above B). The effect on discounted profits, i.e. on the initial scarcity rent, is ambiguous. Both cases (increase or decrease of the initial scarcity rent) can arise. However, we can state the following lemma:

Lemma 5. $\forall \bar{Z} > \theta_e X_e^0 + Z^0$,

$$\frac{d\lambda_e^0}{d\bar{Z}} < 0$$

if and only if

$$\frac{D(p(t^s))}{D(p(0))} \frac{\theta_d}{\theta_e} + (\theta_d - \theta_e) \frac{\mu^0}{\lambda_e^0} > 1.$$

In order to understand this lemma, we can apply backward reasoning. The exhaustible-resource price path can be described by the equation $p_{-t} = c_e + (p^s - c_e)e^{-rt}$, with the convention that extraction ends at date 0 and starts at date $-t^s$. This price is represented by the grey solid line (line L_A) in Figure 4.

¹²Which can be written as $p_d(t) = c_d + (p^s - c_d)e^{rt}$.

With an increase in p^s , the new price is: $\tilde{p}_{-t} = c_e + (p^s - c_e)(1 + \frac{dp^s}{p^s - c_e})e^{-rt}$, which can be rewritten, defining time u by $u = \frac{1}{r} \frac{dp^s}{p^s - c_e}$, as: $\tilde{p}_{-t} = c_e + (p^s - c_e)e^{-r(t-u)}$. So that the new price path \tilde{p}_{-t} is a left shift of the former price path p_{-t} , with some starting date $-\tilde{t}_s$ before $-t^s$. This new price path is represented by the black solid line (line L_B) in Figure 4.

[Insert Figure 4 here.]

As the new price is a translation of the former price, cumulative consumption under price path p_t between dates $-t^s + u$ and 0 is the same as cumulative consumption under price path \tilde{p}_t between dates $-t^s$ and $-u$ (X_A and $X_B = X_A$ in Figure 4). The quantity consumed over price path \tilde{p}_t between dates $-u$ and 0 is equal to $\delta_1 X_B = D(p^s)u$. The quantity consumed over price path p_t between dates $-t^s$ and $-t^s + u$ is equal to $\delta X_A = D(p_{-t^s})u$. As the total quantity consumed over both price paths has to be the same, then it must be the case that the quantity consumed between date $-\tilde{t}_s$ and $-t_s$ is equal to $\delta X_A - \delta_1 X_B = (D(p_{-t^s}) - D(p^s))u$, so that $\tilde{t}_s - t_s = \frac{(D(p_{-t^s}) - D(p^s))u}{D(p_{-t^s})}$. As such, the duration of the extraction with the new price path is:

$$\tilde{t}^s = t^s + \frac{D(p_{-t^s}) - D(p^s)}{D(p_{-t^s})} \frac{dp^s}{r(p^s - c_e)}. \quad (4.1)$$

The increase in the switch price entails higher prices over the whole price path as well as a longer duration of extraction. This longer duration depends on the size of the demand drop between dates $-t^s$ and 0. The overall effect on the initial price is nevertheless positive, as illustrated in Figure 4 (dp_{-t^s}). The effect on the initial scarcity rent on the other hand is ambiguous. Let μ^s and λ_e^s be the shadow-cost values at the time of the switch, and $\lambda_e^{-t^s}$ the scarcity rent at date $-t^s$. We have that $\lambda_e^{-t^s} = (p^s - \mu^s - c_e)e^{-rt^s}$. If there was a rise in p^s only, and not in μ^s , the effect on $\lambda_e^{-t^s}$ would be $\delta_1 \lambda_e^{-t^s}$, which is equal to: $\delta_1 \lambda_e^{-t^s} = dp^s e^{-rt^s} - r(p^s - \theta_e \mu^s - c_e)e^{-rt^s} dt^s$. It can be rewritten, using equation 4.1 with $dt^s = \tilde{t}^s - t^s$:

$$\frac{\delta_1 \lambda_e^{-t^s}}{\lambda_e^{-t^s}} = \left(\frac{D(p^s)}{D(p_{-t^s})} \frac{\lambda^s}{\lambda^s + \theta_e \mu^s} + \frac{\theta_e \mu^s}{\lambda^s + \theta_e \mu^s} \right) \frac{dp^s}{\lambda^s}.$$

This first derivative is obviously positive, but of smaller size than $\frac{dp^s}{\lambda^s}$ as extraction duration¹³ rises with p^s . This effect can be seen as a “competition effect”, this would be the only effect had the carbon tax on the exhaustible resource remained unchanged but the switch price rose at the switch date. With this effect only, the scarcity rent would be put upward. This virtual new scarcity rent corresponds to $\lambda_e^{comp}(t)$, with $\lambda_e^{comp}(0) \equiv \lambda_e^{comp}$ on Figure 5. There is an additional effect of the fall in the ceiling through the rise in the shadow cost of pollution $d\mu^s$, and thus of the tax on R_e , leading to a lower scarcity rent at date $-t^s$ equal to:

$$\frac{\delta_2 \lambda_e^{-t^s}}{\lambda_e^{-t^s}} = -\theta_e \frac{d\mu^s}{\lambda^s}.$$

This second derivative is a “capture effect”: this would be the only effect had the switch price remained unchanged but the carbon tax on the exhaustible resource risen at the switch date. This effect is obviously negative: an increase in the tax on the exhaustible resource, without any increase in the switch price, reduces the initial scarcity rent. The new scarcity rent, λ_e^{0*} , is thus lower than the value of the scarcity rent λ_e^{comp} resulting from the “competition effect” only.

Of course the change in p^s and the change in μ^s are linked by $dp^s = \theta_d d\mu^s$, so that the total effect is:

$$\frac{d\lambda_e^{-t^s}}{\lambda_e^{-t^s}} = \frac{\theta_e \mu^s}{\lambda^s + \theta_e \mu^s} \left(\frac{\theta_d}{\theta_e} \frac{D(p^s)}{D(p^{-t^s})} + (\theta_d - \theta_e) \frac{\mu^s}{\lambda^s} - 1 \right) \frac{d\mu^s}{\mu^s}.$$

Lemma 5 follows.

We now return to the former notation, with the extraction of R_e starting at date 0 and ending at date t^s . Note that if $(\theta_d - \theta_e) \frac{\mu^0}{\lambda_e^0} > 0$, a sufficient condition for the Grey paradox to hold is that $\frac{D(p(t^s))}{D(p(0))} \frac{\theta_d}{\theta_e} > 1$. When $D(p(t^s)) = D(p(0))$, it is always the case that $\frac{D(p(t^s))}{D(p(0))} \frac{\theta_d}{\theta_e} > 1$. This very simple example underlines the role of the elasticity of demand in the outcome. Without more information on this elasticity, we can, however, show that following propositions hold:

Proposition 6. *If R_e becomes exhausted and R_d is used, and if the elasticity*

¹³If the increase in the final price does not entail longer extraction duration, i.e if the elasticity of demand is zero, this effect is equal to $\frac{dp^s}{\lambda^s}$.

of demand is small enough, tightening the carbon ceiling increases the profit of R_e owners.

$\exists \epsilon^*$ such that:

$$\left\{ \forall p, -\frac{D'(p)p}{D(p)} \leq \epsilon^* \text{ and } \bar{Z} > \theta_e X_e^0 + Z^0 \right\} \implies \frac{d\lambda_e^0}{d\bar{Z}} < 0.$$

The formal proof appears in Appendix A.2. The intuition is the following. As the elasticity of demand is small, the effect of an increase in the switch price p^s is high. If the elasticity is low, the duration of extraction is not increased much. The lower the elasticity of demand, the greater is the “competition effect”, which overcomes the “capture effect”.

Proposition 7. *If R_e becomes exhausted and R_d is used, and if the pollution content of R_e is small enough compared to that of R_d , tightening the carbon ceiling increases the profit of R_e owners.*

$\exists \eta^*$, with $0 < \eta^* < 1$, such that:

$$\left\{ \frac{\theta_e}{\theta_d} \leq 1 - \eta^* \text{ and } \bar{Z} > \theta_e X_e^0 + Z^0 \right\} \implies \frac{d\lambda_e^0}{d\bar{Z}} < 0.$$

Proof. Straightforward from Lemma 5. □

If the pollution content of R_e is low enough compared to that of R_d , then R_e has a considerable advantage with respect to its direct competitor (R_d), and the owners of R_e benefit from carbon regulation.

Consider now the extreme case in which $c_e = c_d$. With carbon regulation, the two resources are extracted simultaneously, so that $\lambda_e^0 = (\theta_d - \theta_e)\mu^0$. The initial scarcity rent is positive and increases with carbon regulation. We show in the next proposition that this result continues to hold even with different extraction costs, if they are close enough to each other.

Proposition 8. *If R_e becomes exhausted and R_d is used, and the extraction cost of the exhaustible resource is close enough to that of the dirty backstop, tightening the carbon ceiling increases the profits of R_e owners.*

$\forall \bar{Z} > \theta_e X_e^0 + Z^0, \exists c^* < c_d$ such that:

$$c_d \geq c_e \geq c^* \implies \frac{d\lambda_e^0}{d\bar{Z}} < 0.$$

The formal proof appears in Appendix A.3.

A different result pertains if the exhaustible resource is scarce enough. In this case, the competition effect, at given μ^s, λ^s, p^s and dp^s , increases with the scarcity of the exhaustible resource. The greater extraction duration, which results from higher prices, falls as the total stock of the exhaustible resource falls. The “competition effect” is consequently higher than the “capture effect” when the exhaustible resource is scarce enough.

Proposition 9. *If R_e becomes exhausted and the backstop is used, and R_e is scarce enough, tightening the carbon ceiling increases the profits of R_e owners.*

$\forall \bar{Z} > Z^0$, there exists X^* such that:

$$X_e^0 < X^* \implies \frac{d\lambda_e^0}{d\bar{Z}} < 0.$$

The formal proof appears in Appendix A.4.

For general demand functions, it is not clear if the Grey paradox is more likely to occur for low or high values of the carbon ceiling. We nevertheless do have the following result:

Proposition 10. *For concave or linear demand functions, the profits of R_e resource owners cannot exhibit a U-shape as the carbon ceiling is tightened, i.e the Grey Paradox cannot pertain when making the carbon regulation more stringent, if it does not take place under less strict carbon regulation.*

$\forall Z^1, Z^2$ such that $Z^1 > \theta_e X_e^0 + Z^0, Z^2 > \theta_e X_e^0 + Z^0$ and $Z^1 > Z^2$, if $D'' < 0$,

$$\frac{d\lambda_e^0}{d\bar{Z}} \Big|_{\bar{Z}=Z^1} > 0 \implies \frac{d\lambda_e^0}{d\bar{Z}} \Big|_{\bar{Z}=Z^2} > 0.$$

The formal proof is contained in Appendix A.5.

With a concave demand function, the demand decreases more and more when the energy price increases. Consider a series of same-sized decreases

of the carbon ceiling. As shown above, falls in the ceiling lead to higher and higher switch prices. But increases of this switch price are smaller and smaller due to the increased fall of the demand. Thus, the “competition effect” increases but decelerates when the carbon ceiling falls. As the carbon ceiling is lowered, the date of switch is more and more postponed, thus for a given switch price, an increase of the tax at the date of switch reduces more and more the initial scarcity rent. In other words, the “capture effect” becomes more and more important. It comes that the “competition effect” cannot dominate the “capture effect” (no Grey Paradox occurs) when making the carbon regulation more stringent, if this is not the case under less strict carbon regulation.

Remark 11. Some technological options such as carbon capture allow keeping on using fossil fuels without drastically increasing carbon concentration. Assuming such a technology available at constant marginal cost c_{ccs} , the Grey Paradox can still occur: previous results from Lemma 5 to Proposition 10 continue to hold. Allowing for carbon capture leads to introduce a second clean backstop: the dirty backstop whose emissions are captured, at price $c_d + \theta_d c_{ccs}$. As demonstrated in the literature (Chakravorty et al. 2006, Lafforgue et al. 2008), it is not optimal to capture CO_2 before the ceiling binds. If $X_e^0 < \frac{\bar{Z} - Z^0}{\theta_e}$, both polluting resources are used to get to the ceiling. The dirty backstop with carbon capture is used at the ceiling if and only if $c_b \geq c_d + \theta_d c_{ccs}$. In this case, i.e if $c_b \geq c_d + \theta_d c_{ccs}$ and $X_e^0 < \frac{\bar{Z} - Z^0}{\theta_e}$, the extraction path is as described by equations 3.1-3.4, with c_b replaced by $c_d + \theta_d c_{ccs}$.

4.2 The exhaustible resource is more expensive to extract than the dirty backstop

If the exhaustible resource is more expensive to extract than the dirty backstop, $c_e > c_d$, the exhaustible resource is not exhausted if there is no carbon regulation. Moving from a situation where the exhaustible resource is not used to a situation where it is exhausted, profits must increase at some point.

We assume that resource extraction is as described in Case *B1* ($c_b > \frac{\theta_e c_d - \theta_d c_e}{\theta_e - \theta_d}$, and $X_e^0 < \min(X^*, \frac{\bar{Z} - Z^0}{\theta_e})$).

[Insert Figure 6 here.]

Case $B1$ is described in Figure 6. The bold curves represent fossil-fuel prices and the medium-thick curve the scarcity rent. The initial price paths are given by the plain curves. The dotted curves represent these prices after a fall in the ceiling. The segment AB represents the tax by unit of R_e at the switch date from R_d to R_e . The symbol “*” indicates the new values after a fall in the ceiling.

Tightening the ceiling increases the carbon tax, brings forward the date of switch to R_e and the date the ceiling binds, and increases the profits of R_e owners. First remark that the consumption of R_e is unchanged, as it is exhausted: hence its price when it starts to be used is unchanged and is denoted by p^s . It immediately follows that the value of the tax at this date is unchanged ($AB = A^*B^*$), and so current value profits at that date are the same. If the switch date was postponed, the global consumption of R_d would increase, which would violate the ceiling constraint. Thus the switch date is brought forward (from A to A^*), and the length of the period of consumption of R_e is unchanged. The new price paths for R_e and R_d are represented by the bold dotted line.

If these new price paths were only due to an increase of R_d price, and if the tax on R_e was unchanged, then R_e before-tax price would be equal to $c_e + \lambda_e^{comp}(t) = p_e^*(t) - \theta_e \mu^0 e^{rt}$, represented by the top medium-thick dotted line. The change in the initial tax differential would be $\theta_d d\mu^0$. If this “competition effect” was the only effect of tightening the ceiling, the initial scarcity rent would increase from λ_e^0 to λ_e^{comp} . But this effect is reduced by a “capture effect”, as the tax also applies to R_e : the change in the initial tax differential is not $\theta_d d\mu^0$ but $(\theta_d - \theta_e) d\mu^0$. The “capture effect” never dominates the “competition effect”.

Prices of R_e and R_d cross at the unchanged value, p^s , and the switch date is brought forward, it follows that the differential in initial prices must decrease, so the initial scarcity rent must increase. Thus, the following proposition holds:

Proposition 12. *As long as both resources are used and R_e is fully used, tightening the ceiling constraint increases the scarcity rent of R_e .*

If $c_b > \frac{\theta_e c_d - \theta_d c_e}{\theta_e - \theta_d}$ and $X_e^0 < \min(X^*, \frac{\bar{Z} - Z^0}{\theta_e})$, then:

$$\frac{d\lambda_e^0}{d\bar{Z}} < 0.$$

Remark 13. Assuming that a carbon capture technology is available at constant marginal cost c_{ccs} , the Grey Paradox still occurs as long as both polluting resources are used to get to the ceiling. Proof appears in Appendix A.6.

5 Extensions

In this section, we discuss three main extensions to the model: *(i)* a two-sector economy where the exhaustible resource has a comparative advantage in one sector; *(ii)* an economy with two exhaustible polluting resources and a dirty and a clean backstop; and *(iii)* increasing extraction costs with cumulative extraction. These main extensions will be developed in two companion papers.

(i) A two-sector economy. The stylized facts regarding fossil-fuel consumption suggest the following: different fossil fuels are used simultaneously, with some sectoral specialization, as oil is used mostly in the transport sector, and coal in power generation and industry. Whereas coal, oil and gas respectively represent 29%, 31% and 21% of the total primary energy demand in 2011, coal share equals 47% in power generation, where oil and gas shares are respectively equal to 6% and 23%. In industry, coal represents 29% of the energy supply, oil 13%, and gas 20%. In transport sector, oil represents 93%, coal and gas shares being negligible (IEA 2013). This pattern can be explained by cost heterogeneity across the different resources according to their sectoral use. In the baseline model, following our assumption of constant extraction costs, resources cannot be used simultaneously. This baseline setting can be easily amended to yield a more plausible extraction path.

Consider an economy close to that in Chakravorty & Krulce (1994) composed of two sectors, transport and power generation. The sectoral energy demands are separable. The dirty backstop and the exhaustible resource are perfect substitutes in the power-generation sector, but the dirty backstop must

be transformed into fuel at a positive cost to be used in the transport sector, contrary to the exhaustible resource that can be used at the same cost in both sectors. Once transformed, fuel from the dirty backstop is a perfect substitute for the exhaustible resource in the transport sector. This conversion cost can be seen as the cost of the coal-to-liquid process, for example. Depending on parameter values, simultaneously extracting the dirty backstop and the exhaustible resource may be optimal, the exhaustible resource tending to be used primarily in the transport sector. However, resources are never simultaneously used within a sector. The price of the exhaustible resource is the same in both sectors. The price of the dirty backstop in the transport sector is obtained from an upward vertical translation of its price in the power sector. As in the baseline model, energy prices net of delivery cost will increase at the social discount rate, and tightening the carbon ceiling translates sectoral energy prices horizontally to the left.

It is obvious that introducing a global carbon ceiling is equivalent to putting two sector-specific carbon ceilings over cumulative sectoral emissions, equal to the optimal sectoral amounts of cumulated emissions given the global ceiling. In addition, tightening the carbon ceiling leads to a lower cumulative amount of emissions in each sector. Finally, tightening carbon regulation in the two-sector economy can be seen as imposing tighter *sectoral ceilings*.

In some cases, the profit effects of tightening regulation are in the same direction in both sectors, leading to unambiguous effect on global profits. First, consider the case where, in one sector, only the exhaustible resource is used. It is immediate that profits in this sector will fall as carbon regulation is tightened. If, in the other sector, the exhaustible resource is never used (i.e. if resources are fully sector-specialized), tightening carbon regulation reduces the global profits of exhaustible-resource owners. Assume that, in the other sector, the resource is used in competition with the dirty backstop: tightening carbon regulation will increase profits in this sector under the conditions given above. However, the impact on global profits is ambiguous: the lower profits in the first sector may or may not be compensated by any potential gains in the other sector. These cases require more analysis. Second, if the exhaustible

resource is not used alone in one sector, and if both polluting resources are used at least in one sector, the results from the baseline model still hold.

(ii) *Multiple polluting exhaustible resources.* A second limitation of our model, compared to the stylized facts, is the assumption of a unique exhaustible polluting resource. In real life, oil and gas are available in addition to abundant coal, and each fossil fuel has different resource grades, which are more or less polluting and expensive (see Table 1). The extension consists in introducing a second exhaustible polluting resource. In this new setting, there are four energy resources, perfect substitutes in demand: two exhaustible polluting resources, an abundant dirtier resource and an abundant clean resource.

Resources are optimally extracted in sequential order without any stop-and-go in resource extraction. If the three polluting resources are used, a number of different cases can occur. The dirty backstop can be used in the first, second or third position, or not used at all. Both exhaustible resources can be exhausted or not. We consider only the cases where the three polluting resources are used, with one resource at least being exhausted. Tightening the carbon ceiling increases profits of owners of any exhausted resource that is used after the dirty backstop. We say that two resources are in direct competition if one of these two resources is directly extracted after the other. If an exhaustible resource is used before the dirty backstop but is in direct competition over time with it, the previous propositions regarding the profits associated with this resource continue to hold.¹⁴ If an exhaustible resource is used before the dirty backstop but is not in direct competition over time with it, the impact of tightening the carbon cap on the profits of the owners of this specific resource is unclear.

With more than one exhaustible resource, a new question arises: do owners of a more polluting grade of resource necessarily lose more or win less than the owners of a less-polluting resource? If the exhaustible resource that is in direct competition over time with the dirty backstop is the least polluting of the two exhaustible resources, or is more polluting but only by little, the owners of this resource will win more or lose less from tighter carbon regulation than the

¹⁴Except for Proposition 9.

owners of another exhaustible resource that is not in direct competition with the dirty backstop.

The intuition is that the “competition effect” for the resource that is in direct competition with the dirtier resource increases more after tighter carbon regulation than the “competition effect” for that which is not in direct competition with it, if the pollution contents of both exhaustible resources are close enough. This result illustrates the importance of extraction costs, and not only pollution content, in assessing the benefits and losses from tighter carbon regulation. Applying this result to the transport sector where inshore conventional oil (oil extracted in Saudi Arabia for instance) competes with offshore conventional oil (which is more polluting and more expensive: oil extracted in the UK or Norway for instance) and unconventional oil (the most polluting and the most expensive type of oil, such as that from bituminous sands extracted in Canada), it follows that owners of offshore oil may benefit more or lose less from tighter carbon regulation than owners of inshore oil, despite the fact that offshore conventional oil is more polluting than inshore conventional oil.

(iii) Increasing extraction costs with cumulative extraction. We here discuss how the assumption of constant marginal extraction costs can be relaxed. Extraction costs are increasing with cumulative extraction: this reflects the rising cost of increasingly deep mining or less pressure in oil wells, for example, or simply reflects the fact natural resources come in more or less costly different grades (Heal 1976, Solow & Wan 1976, Kemp & Long 1980, Lewis 1982). In our framework, as cheaper grades will be exploited before more expensive grades, as in Herfindahl (1967), these grades can be seen as a unique resource whose extraction becomes increasingly costly as it becomes scarcer and scarcer. The analysis can easily be extended to take account of this increasing pattern of costs. Assume that the extraction costs for the exhaustible resource depend on cumulative extraction, and that there exists a series of thresholds over which marginal extraction costs jump, although they remain constant between two thresholds. Let the dirty-backstop extraction cost be constant.

We revisit the previous setting with multiple exhaustible resources, and

assume that the pollution content of the different exhaustible resources is the same. Each resource grade has a specific scarcity rent which increases at the rate of the discount rate, scarcity rents being ordered in the reverse order of extraction costs. Market prices still rise over time but the overall price (including all the grades) rises less and less steeply. Determining the impact of a more stringent carbon ceiling on the profits of the owner of this resource comes down to analyzing the impact on the joint profits related to the different grades. Three general cases can occur: the resource is used before the dirty backstop, after, or before and after.

From previous extension, we immediately get the following results. If the resource is used before, or before and after, the dirty backstop, tightening carbon regulation will increase profits if the elasticity of demand is low enough, or if the pollution content of this resource is low enough, or if the minimum extraction cost is close enough to that of the dirty backstop, or if this resource is scarce enough. If the dirty backstop is used before the other polluting resource, the profits associated with each grade of this resource will increase, and so overall profits will also rise.

6 Conclusion

This paper has cast some light on the distributional effects of carbon taxation, showing that the owners of a carbon-emitting resource may benefit from carbon taxation if a dirtier abundant resource is also used, even if the tax revenues are not redistributed. In particular, when this resource is cheaper to extract than the dirtier resource, tightening carbon regulation increases the profits of the exhaustible-resource owners if any of the following hold *(i)* its demand elasticity is low enough; *(ii)* its extraction cost is close enough to that of the dirty backstop; *(iii)* its pollution content is low enough (compared to that of the dirty backstop); or *(iv)* its initial stock is small enough. When the exhaustible resource is more expensive to extract than the dirty backstop, tightening carbon regulation increases the profits of its owners. Introducing a carbon capture technology from the stock or the flow of CO_2 , with constant

marginal cost, does not change these results. Our model can easily be amended to take account of the sectoral specialization of resources, increasing extraction costs and the existence of multiple carbon-emitting exhaustible resources.

Conventional oil is likely to be in competition over time with unconventional oil, which is both abundant (oil shales, oil sands-based synthetic crudes and derivative products, coal-based liquid supplies, biomass-based liquid supplies and liquids arising from the chemical processing of natural gas) and more polluting. In our model, conventional-oil owners may benefit from carbon taxation. The same remark holds for natural-gas producers, since gas is in competition with more polluting resources such as coal.

Our results lead us to reconsider the debate over compensation for losses in oil-export revenues induced by carbon taxation, as claimed for instance by OPEC countries. Major coal or unconventional-oil exporters are likely to remain insensitive to pro-mitigation arguments as long as their losses are not at least partially compensated. Canada, for instance, withdrew from the Kyoto Protocol in 2011 after the boom in oil sands. Conventional oil and gas exporters, mostly OPEC-Gulf countries, may be more easily convinced of the necessity of carbon regulation if it can be shown that they may directly benefit from carbon taxation.

Technological progress in green technologies is an ongoing topic in the climate-change mitigation debate. If the clean-backstop cost falls over time, the likelihood of the Grey Paradox would increase. Since a fall in the ceiling brings forward the use of the clean backstop, its starting price would increase as the carbon cap falls, making this resource less competitive (compared to polluting resources).

Resource profits can be used in R&D to reduce extraction costs or to explore the earth's crust to find new deposits. Such activities would clearly exacerbate the impact of a fall of the carbon ceiling on total profits in our model.

A final remark is that OPEC domestic markets represented 9.8% of world oil consumption and 23.3% of their own production in 2011. They amount to 12.8% of world gas consumption and 69.8% of their own production (BGR

2012). OPEC countries can use strategies, for instance exempting their domestic market from the tax, to increase their profits or reduce their losses. Allowing for such strategies would clearly make the emergence of the Grey Paradox more likely.

A Appendix

A.1 Proof of Lemma 2

Proof. Let first look at the conditions under which only R_d is used to reach the ceiling (Case B3). First note that if R_d is being used when the ceiling is reached, then R_e is never used, as if $c_e + (\theta_e \mu^0 + \lambda_e^0) e^{r\underline{t}} > c_d + \theta_d \mu^0 e^{r\underline{t}}$ then it implies that for all $t \leq \underline{t}$, $c_e + (\theta_e \mu^0 + \lambda_e^0) e^{rt} > c_d + \theta_d \mu^0 e^{rt}$ (given that $c_d < c_e$). R_d is being used when the ceiling is reached if and only if at date \underline{t} , defined by $c_d + \theta_d \mu^0 e^{r\underline{t}} = c_b$, R_e price satisfies $p_e(\underline{t}) = c_e + \theta_e \mu^0 e^{r\underline{t}} > c_b$. We can write equivalently that only R_d is used to reach the ceiling if and only if: $c_b \leq \frac{\theta_d c_e - \theta_e c_d}{\theta_d - \theta_e}$. Assume now that $c_b > \frac{\theta_d c_e - \theta_e c_d}{\theta_d - \theta_e}$, then R_e is used at the binding date. The carbon tax τ_t while R_e is being used must satisfy $c_e + \theta_e \tau_t \leq c_d + \theta_d \tau_t$, which can be rewritten $\tau_t \geq \frac{c_e - c_d}{\theta_d - \theta_e}$. The lowest possible price path of R_e is thus $p(t) = c_e + \frac{c_e - c_d}{\theta_d - \theta_e} e^{rt}$. Along this path, the price reaches the backstop price c_b at date T^* defined by $c_e + \frac{c_e - c_d}{\theta_d - \theta_e} e^{rT^*} = c_b$. Then the maximum amount of R_e that can be consumed, if $c_b > \frac{\theta_d c_e - \theta_e c_d}{\theta_d - \theta_e}$, is: $X^* = \int_0^{T^*} D(c_e + \frac{c_e - c_d}{\theta_d - \theta_e} e^{rt}) dt$. Note that X^* does not depend on \bar{Z} . If $X_e^0 > X^*$, R_e is not exhausted. If $X_e^0 > X^*$ and $\bar{Z} > Z^0 + \theta_e X^* \equiv Z^*$, then R_d is used first, an amount X^* of R_e is then used to reach the ceiling, and is not exhausted (Case B2). If $X_e^0 < X^*$ and $\bar{Z} < Z^*$, then only R_e is used to reach the ceiling and R_e is not exhausted (Case B4). If $X_e^0 < X^*$ and $\bar{Z} > Z^0 + \theta_e X_e^0$, then R_d is used first, and R_e is then used to reach the ceiling and is exhausted (Case B1). \square

A.2 Proof of Proposition 6

Proof. We know that $\frac{d\lambda_e^0}{dZ} < 0$ if and only if $\frac{D(p(t^*))}{D(p(0))} \frac{\theta_d}{\theta_e} + (\theta_d - \theta_e) \frac{\mu^0}{\lambda_e^0} > 1$. By the mean-value theorem, there exists a date t_i , satisfying $0 \leq t_i \leq t^s$ such

that $\frac{D(p(t^s))}{D(p(0))} = 1 + \frac{D'(p(t_i))}{D(p(0))}(p(t^s) - p(0)) \geq 1 - \left(-\frac{D'(p(t_i))}{D(p(t_i))}p(t_i)\right)\frac{c_b - c_e}{c_e}$. If $\forall p$, $-\frac{D'(p)}{D(p)}p \leq \frac{c_e(1 - \theta_e/\theta_d)}{c_b - c_e} \equiv \epsilon^*$, then tightening the carbon ceiling increases the profits of R_e owners. \square

A.3 Proof of Proposition 8

Proof. Using Lemma 5, and replacing $(\theta_d - \theta_e)\frac{\mu^0}{\lambda_e^0}$ by $1 - \frac{c_d - c_e}{\lambda_e^0 e^{rt^s}}$, it can be seen that $\frac{d\lambda_e^0}{dZ}$ has the sign of $-\frac{D(p(t^s))}{D(p(0))}\frac{\theta_d}{\theta_e} + \frac{c_d - c_e}{\lambda_e^0 e^{rt^s}}$. We know that $\lambda_e^0 e^{rt^s} > (\theta_d - \theta_e)\mu^0 e^{rt^s}$. We can see that $\mu^0 e^{rt^s}$ does not depend on c_e , as $(\mu^0 e^{rt^s}, \underline{t} - t^s)$ are defined by $\theta_d \int_0^{\underline{t} - t^s} D(c_d + \mu^0 e^{rt^s + ru}) du = \bar{Z} - Z^0 - \theta_e X_e^0$ and $\mu^0 e^{rt^s} e^{r(\underline{t} - t^s)} = c_b - c_d$. And $\mu^0 e^{rt^s}$ is strictly positive for any \bar{Z} . At a given value of $\mu_0 e^{rt^s}$, as $-\frac{D(c_b)}{D(c_e)}\frac{\theta_d}{\theta_e} + \frac{c_d - c_e}{(\theta_d - \theta_e)\mu^0 e^{rt^s}}$ is continuous in c_e , falls with c_e , and is strictly negative for $c_e = c_d$, there exists c^* such that Proposition 8 holds. \square

A.4 Proof of Proposition 9

Proof. The demands $D(p(t^s))$ and $D(p(0))$ are continuous functions of the initial stock X_e^0 . Moreover, $\lim_{X \rightarrow 0} \frac{D(p(t^s))}{D(p(0))} = 1$. As a result, $\forall \epsilon, \exists X^*$ such that $X_e^0 < X^* \implies \left\{ \frac{D(p(t^s))}{D(p(0))} \geq 1 - \epsilon \text{ and } \theta_e X_e^0 + Z^0 < \bar{Z} \right\}$. Take $\epsilon < \frac{\theta_d - \theta_e}{\theta_d}$ and the corresponding X^* , then it is the case that, for $X_e^0 < X^*$, $\frac{d\lambda_e^0}{dZ} < 0$. \square

A.5 Proof of Proposition 10

Proof. From Lemma 5 it can be shown that $\frac{d\lambda_e^0}{dZ}$ has the same sign as $-\theta_d D(p(t^s)) + \theta_e D(p(0)) + r\theta_d \mu^0 \theta_e \int_0^{t^s} D'(p(t))e^{rt} dt$. Thus, when $\frac{d\lambda_e^0}{dZ} = 0$, $\frac{d^2\lambda_e^0}{d^2Z}$ has the sign of: $-\theta_d D'(p(t^s))\lambda_e^0 r e^{rt^s} \frac{dt^s}{dZ} + (\theta_e - \theta_d) D'(p(0))\theta_e \frac{d\mu^0}{dZ} - \theta_d \frac{d\mu^0}{dZ} \theta_e \int_0^{t^s} D''(p(t))r e^{rt} \lambda_e^0 e^{rt} dt$. If $D'' \leq 0$, $-\theta_d \frac{d\mu^0}{dZ} \theta_e \int_0^{t^s} D''(p(t))r e^{rt} \lambda_e^0 e^{rt} dt < 0$, thus $\frac{d^2\lambda_e^0}{d^2Z} \Big|_{\frac{d\lambda_e^0}{dZ}=0} < 0$. \square

A.6 Proof of Remark 13

CO_2 is captured if and only if $c_b \geq \min(c_e + \theta_e c_{ccs}, c_d + \theta_d c_{ccs})$. When CO_2 is captured, R_e is used before the ceiling if and only if $c_e + \theta_e c_{ccs} > \frac{\theta_e c_d - \theta_d c_e}{\theta_e - \theta_d}$. It implies also that $c_e + \theta_e c_{ccs} \leq c_d + \theta_d c_{ccs}$. If used, R_e becomes exhausted,

since the price of a non-exhausted exhaustible resource whose emissions are captured equals $c_e + \theta_e c_{ccs}$, which is lower than the price of the other resources at the ceiling i.e $\min(c_b, c_d + \theta_d c_{ccs})$ thanks to the previous assumption. There exists a quantity $X^*(\bar{Z})$ such that, only R_e is used to get to the ceiling if and only if $X_e^0 \geq X^*(\bar{Z})$. We call \bar{t} the date of switch from R_e to the cheaper clean backstop. Variables $(\lambda_e^0, \mu^0, \underline{t}, \bar{t}, X^*)$ are defined by: (i) $c_e + \lambda_e^0 e^{r\bar{t}} + \theta_e c_{ccs} = \min(c_b, c_d + \theta_d c_{ccs})$; (ii) $c_d + \theta_d \mu^0 = c_e + \lambda_e^0 + \theta_e \mu^0$; (iii) $c_{ccs} = \mu^0 e^{r\underline{t}}$; (iv) $\theta_e \int_0^{\underline{t}} D(c_e + \lambda_e^0 e^{rt} + \theta_e \mu^0 e^{rt}) dt = \bar{Z} - Z^0$; (v) $X^* = \frac{\bar{Z} - Z^0}{\theta_e} + \int_{\underline{t}}^{\bar{t}} D(c_e + \lambda_e^0 e^{rt} + \theta_e c_{ccs}) dt$. X^* is the maximum quantity of R_e that will be used if only R_e is used to get to the ceiling. If $X_e^0 > X^*(\bar{Z})$ then R_d is used, prior to R_e , to get to the ceiling. If capture is used, both resources will be used to get to the ceiling if and only if $X_e^0 > X^*(\bar{Z})$ and $c_e + \theta_e c_{ccs} > \frac{\theta_e c_d - \theta_d c_e}{\theta_e - \theta_d}$.

We show that the scarcity rent of R_e increases as the ceiling is lowered if carbon capture is used and both resources are used to get to the ceiling. With $\lambda^s \equiv \lambda_e(t^s)$ and $\mu^s \equiv \mu(t^s)$, the price path of R_e is given by the following set of equations: (i) $c_e + \lambda^s e^{r(\bar{t}-t^s)} + \theta_e c_{ccs} = \min(c_b, c_d + \theta_d c_{ccs})$; (ii) $c_d + \theta_d \mu^s = c_e + \lambda^s + \theta_e \mu^s$; (iii) $c_{ccs} = \mu^s e^{r(\underline{t}-t^s)}$; (iv) $X_e^0 = \int_{t^s}^{\underline{t}} D(c_e + \lambda^s e^{r(t-t^s)} + \theta_e \mu^s e^{r(t-t^s)}) dt + \int_{\underline{t}}^{\bar{t}} D(c_e + \lambda^s e^{r(t-t^s)} + \theta_e c_{ccs}) dt$. This set of equations defines $(\lambda^s, \mu^s, \underline{t} - t^s, \bar{t} - t^s)$ which do not depend on \bar{Z} . As a result, the price path for R_e is unchanged with \bar{Z} . This implies that emissions from R_e before the ceiling remain the same, so that the cumulative extraction of R_d must decrease as the ceiling is lowered. As the switch price from R_d to R_e does not depend on \bar{Z} either, the duration of extraction of R_d before the ceiling must decrease. The scarcity rent at the date of switch is unchanged, it comes that the initial scarcity rent is higher as the carbon ceiling is lowered. Finally, if both polluting resources are used to get to the ceiling, and R_e becomes exhausted, i.e if $c_b \geq \min(c_e + \theta_e c_{ccs}, c_d + \theta_d c_{ccs})$ and $c_e + \theta_e c_{ccs} > \frac{\theta_e c_d - \theta_d c_e}{\theta_e - \theta_d}$ and $X_e^0 > X^*(\bar{Z})$, tightening the carbon ceiling increases the profits of the exhaustible-resource owners.

References

- Allen, M. R., Frame, D. J., Huntingford, C., Jones, C. D., Lowe, J. A., Meinshausen, M. & Meinshausen, N. (2009), 'Warming caused by cumulative carbon emissions towards the trillionth tonne', *Nature* **458**(7242), 1163–1166.
- Barnett, J., Dessai, S. & Webber, M. (2004), 'Will opec lose from the kyoto protocol?', *Energy Policy* **32**(18), 2077–2088.
- Bartsch, U. & Müller, B., eds (2000), *Fossil Fuels in a Changing Climate*, Oxford University Press, Oxford, UK.
- Bergstrom, T. C. (1982), 'On capturing oil rents with a national excise tax', *American Economic Review* **72**(1), 194–201.
- Bernstein, P. M., Montgomery, W. D., Rutherford, T. F. & Yang, G.-F. (1999), 'Effects of restrictions on international permit trading: The ms-mrt model', *The Energy Journal* **20**(Special Issue), 221–256.
- BGR (2012), Energy resources 2012, Technical report, BGR, German Federal Institute for Geosciences and Natural Resources, Hannover, Germany.
- Brander, J. & Djajic, S. (1983), 'Rent-extracting tariffs and the management of exhaustible resources', *Canadian Journal of Economics* **16**(2), 288–98.
- Burnham, A., Han, J., Clark, C. E., Wang, M., Dunn, J. B. & Palou-Rivera, I. (2012), 'Life-cycle greenhouse gas emissions of shale gas, natural gas, coal, and petroleum', *Environmental Science & Technology* **46**(2), 619–627.
- Chakravorty, U. & Krulce, D. L. (1994), 'Heterogeneous demand and order of resource extraction', *Econometrica* **62**(6), 1445–1452.
- Chakravorty, U., Magné, B. & Moreaux, M. (2006), 'A hotelling model with a ceiling on the stock of pollution', *Journal of Economic Dynamics and Control* **30**(12), 2875–2904.
- Chakravorty, U., Moreaux, M. & Tidball, M. (2008), 'Ordering the extraction of polluting nonrenewable resources', *American Economic Review* **98**(3), 1128–1144.
- Ezzati, A. (1976), 'Future opec price and production strategies as affected by its capacity to absorb oil revenues', *European Economic Review* **8**(2), 107–138.
- Ghanem, S., Lounnas, R. & Brennand, G. (1999), 'The impact of emissions trading on opec', *OPEC Review* **23**(2), 79–112.
- Heal, G. (1976), 'The relationship between price and extraction cost for a resource with a backstop technology', *Bell Journal of Economics* **7**(2), 371–378.
- Herfindahl, O. C. (1967), "Depletion and Economic Theory" In *Extractive Resources and Taxation ed. Mason Gaffney, 63-90*, University of Wisconsin Press.
- Hotelling, H. (1931), 'The economics of exhaustible resources', *Journal of Political Economy* **39**(2), 137–175.
- IEA (2013), 'World energy outlook 2013', *IEA Papers* .
- Karp, L. (1984), 'Optimality and consistency in a differential game with non-renewable resources', *Journal of Economic Dynamics and Control* **8**(1), 73–98.
- Kemp, M. C. & Long, N. V. (1980), 'On two folk theorems concerning the extraction

- of exhaustible resources', *Econometrica* **48**(3), 663–673.
- Lafforgue, G., Magné, B. & Moreaux, M. (2008), 'Energy substitutions, climate change and carbon sinks', *Ecological Economics* **67**(4), 589–597.
- Lewis, T. R. (1982), 'Sufficient conditions for extracting least cost resource first', *Econometrica* **50**(4), 1081–1083.
- Liski, M. & Tahvonen, O. (2004), 'Can carbon tax eat opec's rents?', *Journal of Environmental Economics and Management* **47**(1), 1–12.
- MacAvoy, P. W. (1982), *Crude Oil Prices as Determined by OPEC and Market Fundamentals*, Ballinger Publishing Company, Cambridge, MA.
- McKibbin, W. J., Ross, M. T., Shackleton, R. & Wilcoxon, P. J. (1999), 'Emissions trading, capital flows and the kyoto protocol', *The Energy Journal* **20**(Special Issue), 287–333.
- Persson, T. A., Azar, C., Johansson, D. & Lindgren, K. (2007), 'Major oil exporters may profit rather than lose, in a carbon-constrained world', *Energy Policy* **35**(12), 6346–6353.
- Polidano, C., Jotzo, F., E.Heyhoe, G.Jakeman, K.Woffenden & Fisher, B. (2000), 'The kyoto protocol and developing countries: impacts and implications for mechanism design', Abare research report, Australian Bureau of Agricultural and Research Economics, Canberra.
- Rubio, S. J. & Escriche, L. (2001), 'Strategic pigouvian taxation, stock externalities and polluting non-renewable resources', *Journal of Public Economics* **79**(2), 297–313.
- Solow, R. M. & Wan, F. Y. (1976), 'Extraction costs in the theory of exhaustible resources', *The Bell Journal of Economics* **7**(2), 359–370.
- Strand, J. (2010), 'Optimal fossil fuel taxation with backstop technologies and tenure risk', *Energy Economics* **32**(2), 418–422.
- Tahvonen, O. (1997), 'Fossil fuels, stock externalities, and backstop technology', *Canadian Journal of Economics* **30**(4), 855–874.
- Ulph, A. & Ulph, D. (1994), 'The optimal time path of a carbon tax', *Oxford Economic Papers* **46**(Special Issue), 857–868.
- van der Ploeg, F. & Withagen, C. (2012), 'Too much coal, too little oil', *Journal of Public Economics* **96**(1), 62–77.
- Verleger, P. K. (1982), 'The determinants of official opec crude prices', *The Review of Economics and Statistics* **64**(2), 177–182.
- Wirl, F. (1995), 'The exploitation of fossil fuels under the threat of global warming and carbon taxes: A dynamic game approach', *Environmental and Resource Economics* **5**(4), 333–352.

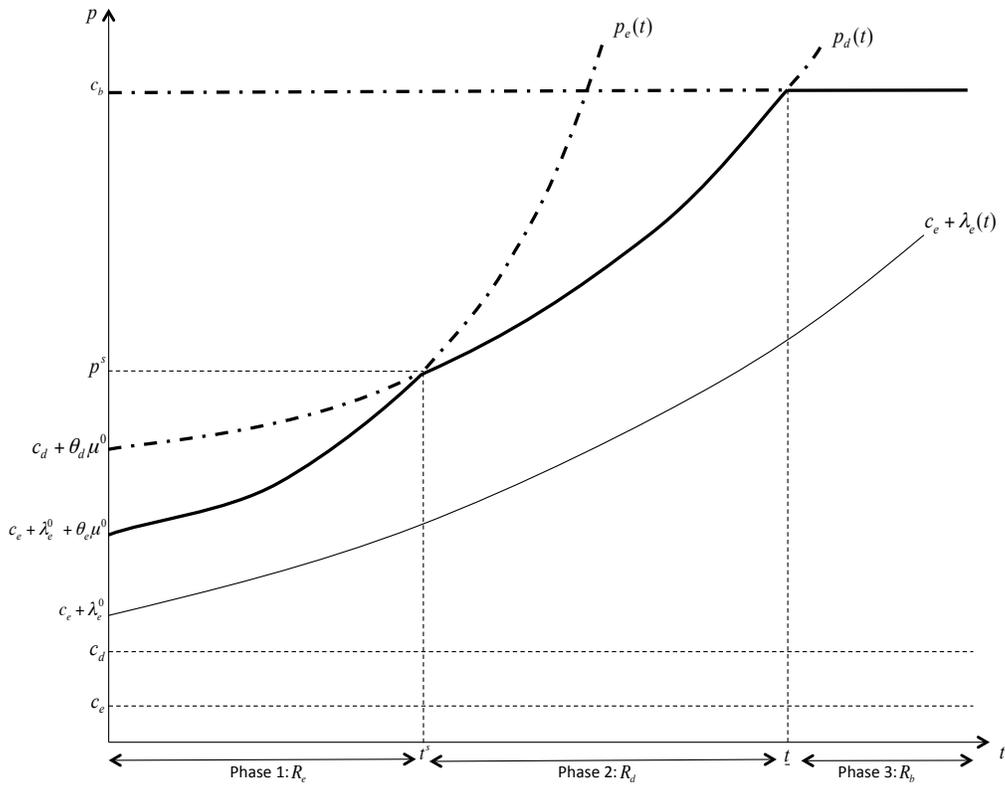


Figure 1: Energy price path (bold solid line) when $c_e < c_d$, Case A1

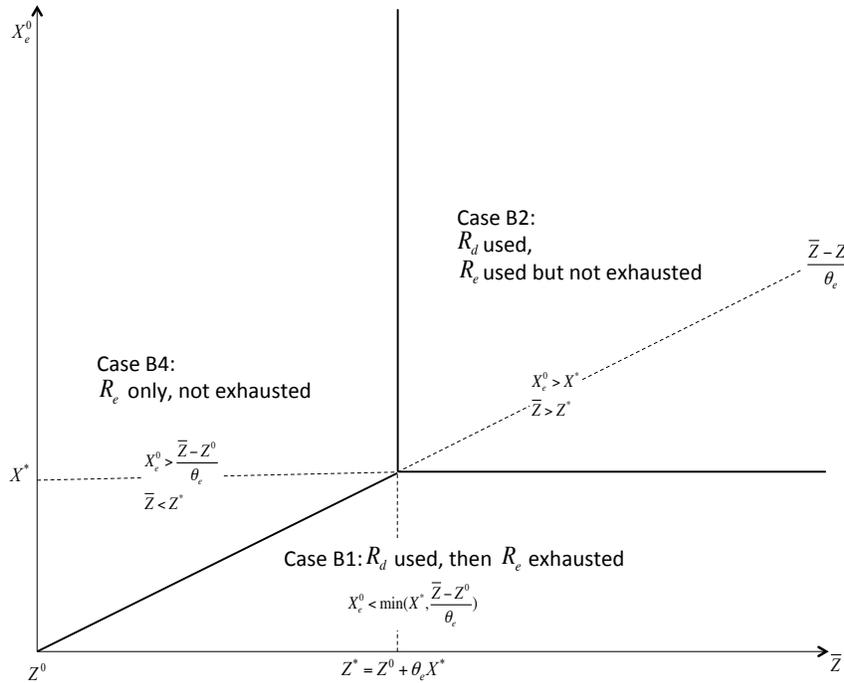


Figure 2: The different cases when $c_e > c_d$ and $c_b > \frac{\theta_d c_e - \theta_e c_d}{\theta_d - \theta_e}$

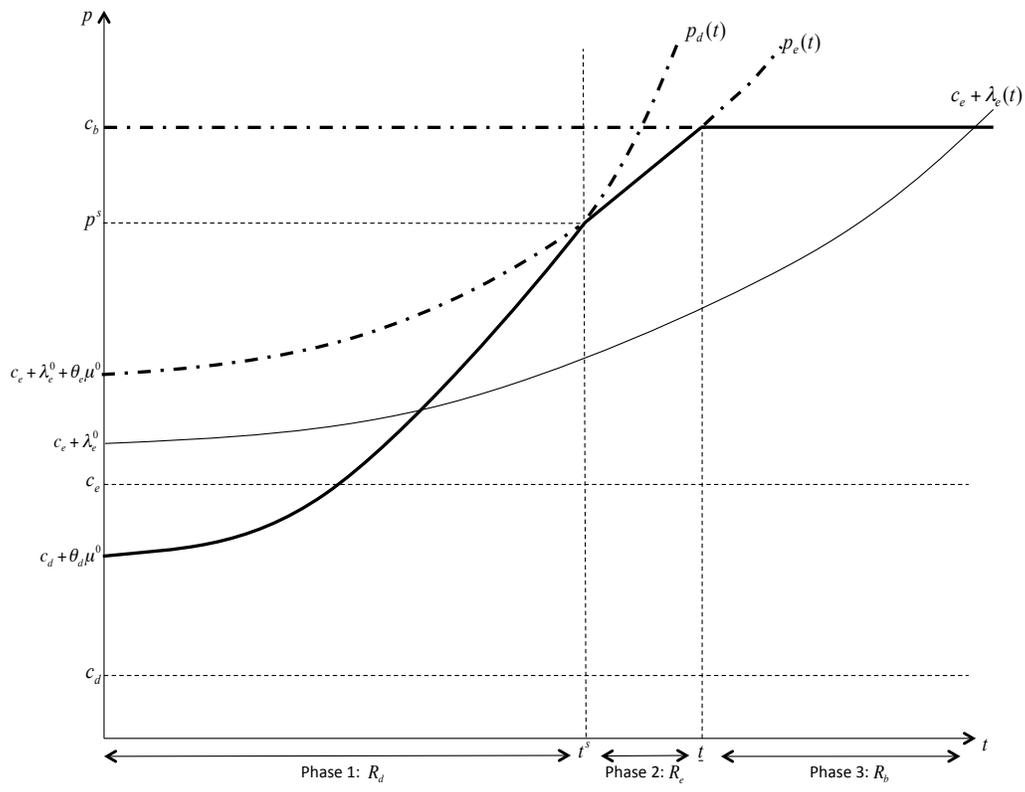


Figure 3: Energy price path (bold solid line) when $c_e > c_d$, Case B1

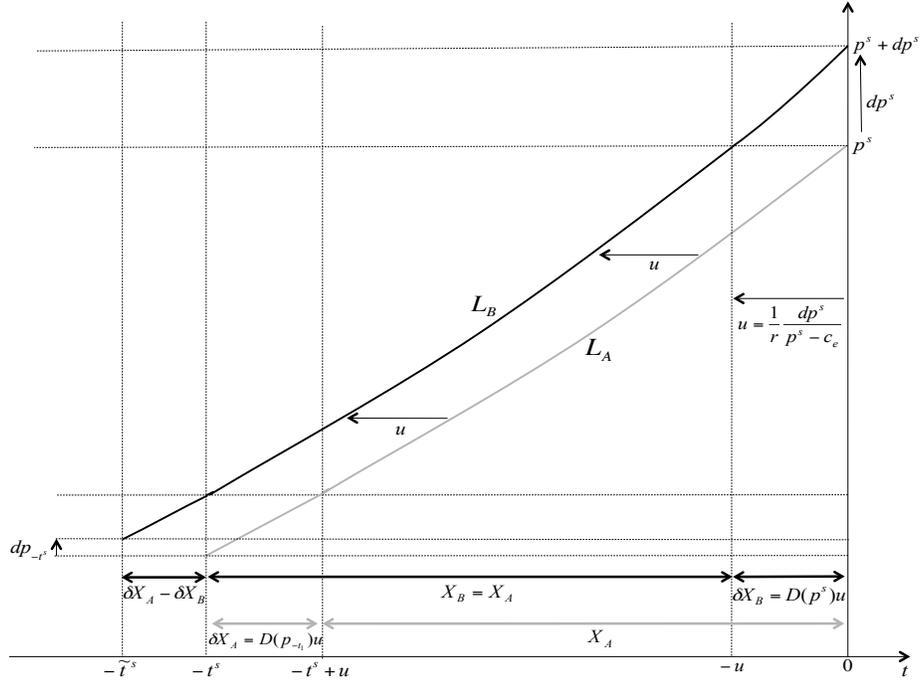


Figure 4: The change in R_e price path as the ceiling is lowered, $c_e < c_d$, Case A1

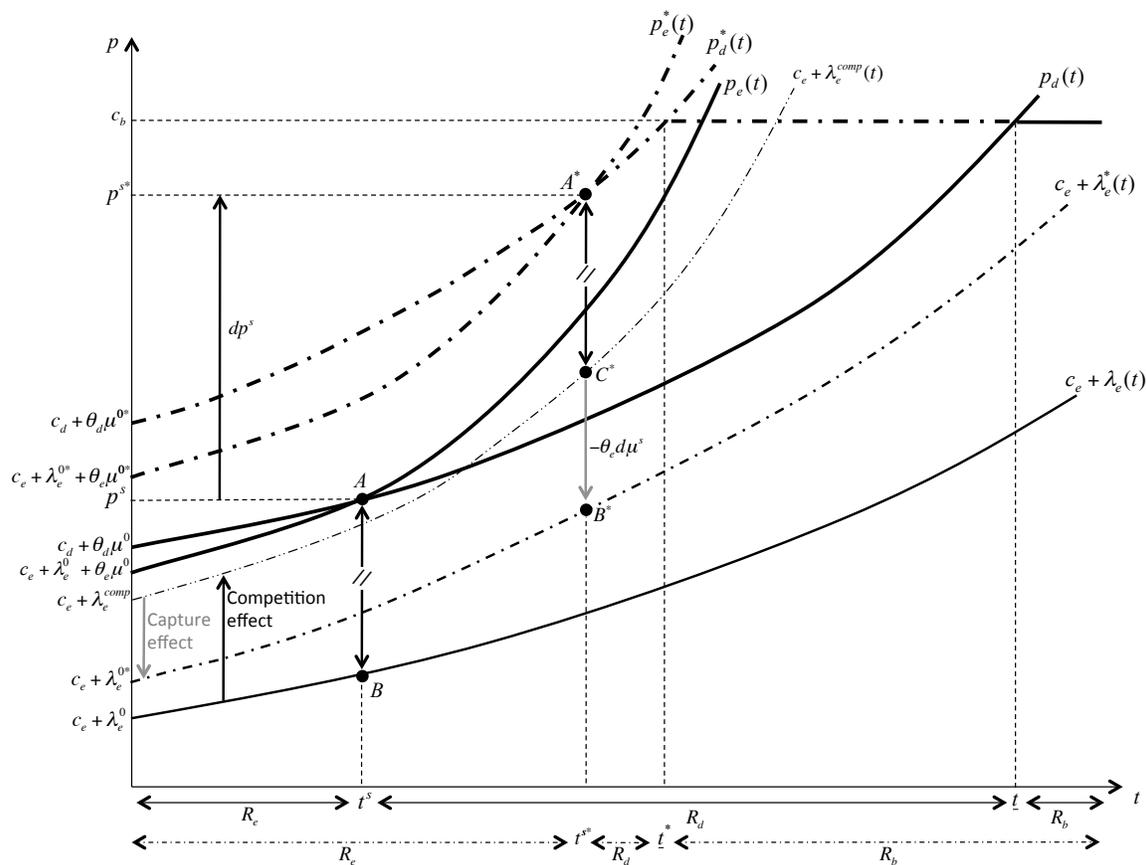
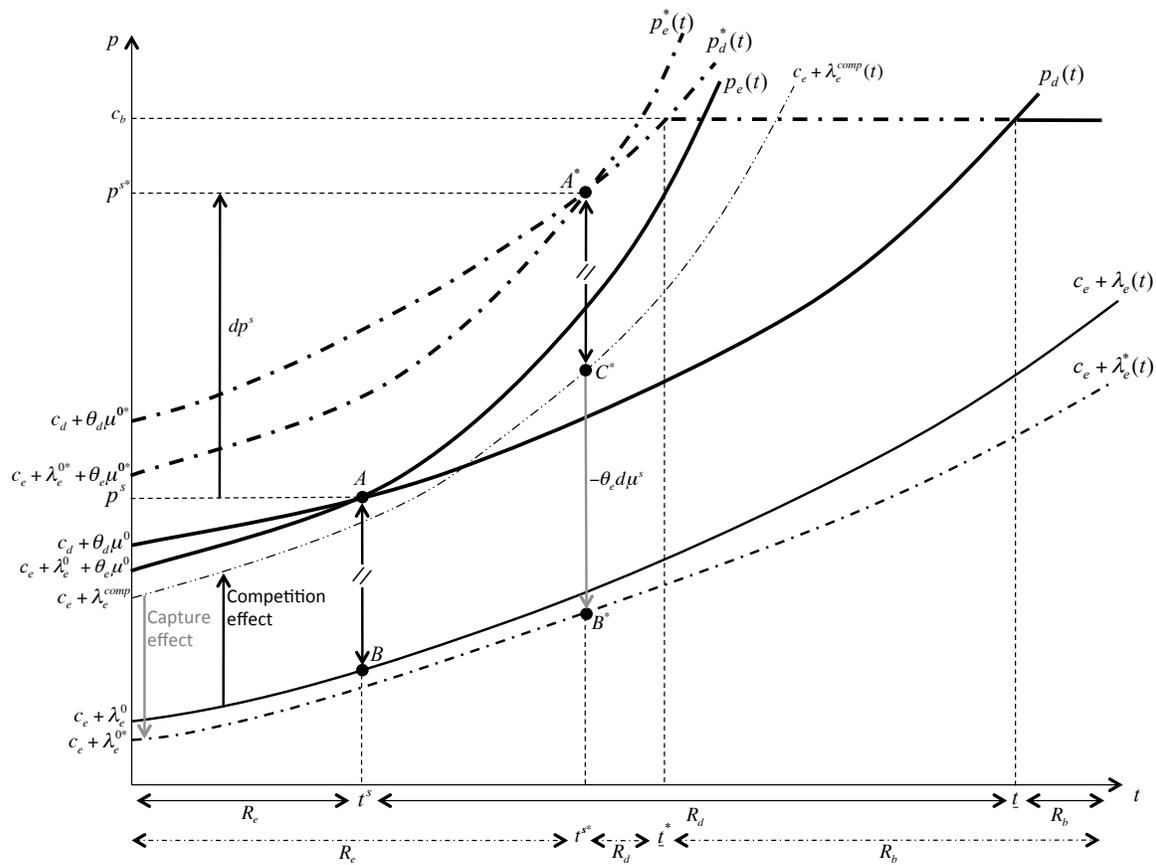


Figure 5: Price paths as the carbon ceiling is lowered, $c_e < c_d$, Case A1. Occurrence of the Grey Paradox (bottom panel) or not (top panel)

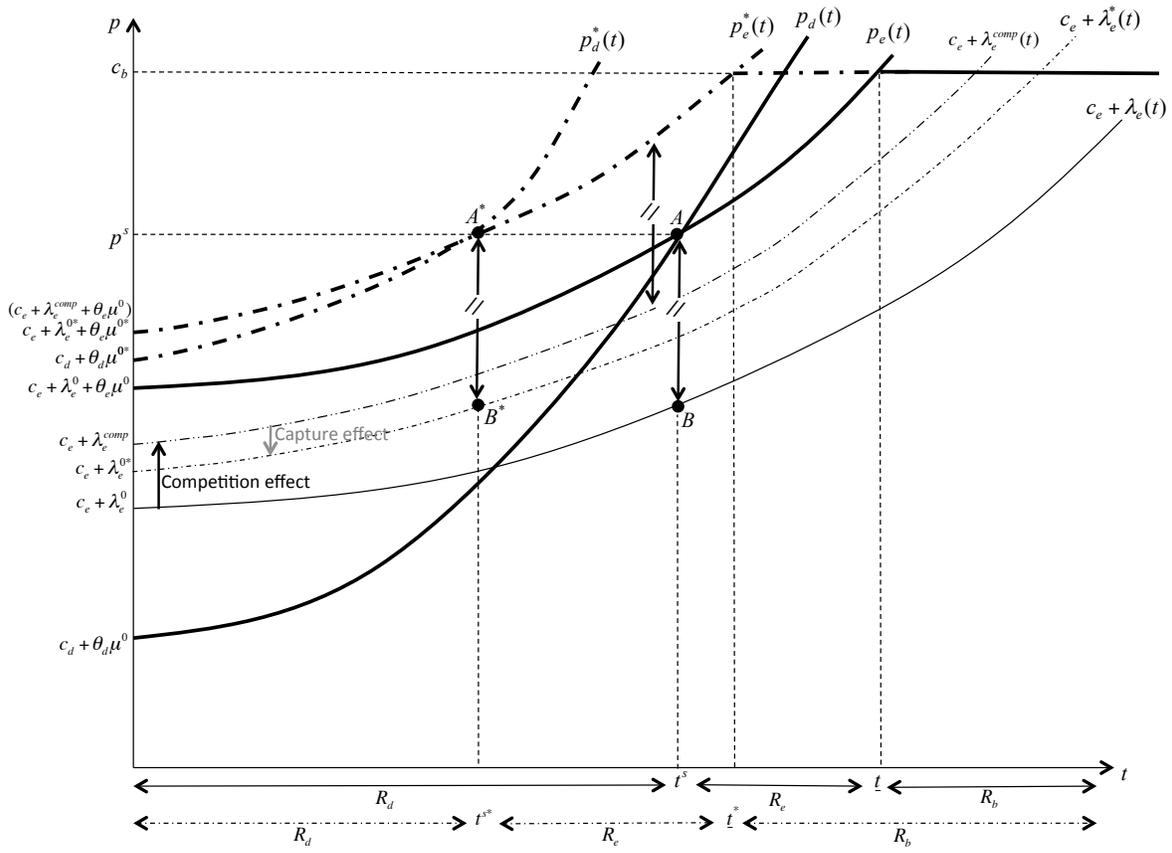


Figure 6: Price paths as the carbon ceiling is lowered, $c_e > c_d$, Case $B1$